

Answer Key 2

Page 102: For 3-9, just find the limit. 12-34, 37.	Page 125: 35-38, 47, 48, 51, 53, 55, 56, 75
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3. $\lim_{x \rightarrow 5} (4x^2 - 5x) = \lim_{x \rightarrow 5} 4x^2 - \lim_{x \rightarrow 5} 5x =$ $4 \lim_{x \rightarrow 5} x^2 - 5 \lim_{x \rightarrow 5} x = 4 \cdot 5^2 - 5 \cdot 5 = 75$	4. $\lim_{x \rightarrow -3} (2x^3 + 6x^2 - 9) =$ $2 \lim_{x \rightarrow -3} x^3 + 6 \lim_{x \rightarrow -3} x^2 - \lim_{x \rightarrow -3} 9 =$ $2(-3)^3 + 6(-3)^2 - 9 = -54 + 54 - 9 = -9$
5. $\lim_{v \rightarrow 2} (v^2 + 2v)(2v^3 - 5) =$ $\lim_{v \rightarrow 2} (v^2 + 2v) \lim_{v \rightarrow 2} (2v^3 - 5) =$ $\left[\lim_{v \rightarrow 2} v^2 + 2 \lim_{v \rightarrow 2} v \right] \left[2 \lim_{v \rightarrow 2} v^3 - \lim_{v \rightarrow 2} 5 \right]$ $[2^2 + 2 \cdot 2][2 \cdot 2^3 - 5] = 8 \cdot 11 = 88$	6. $\lim_{t \rightarrow 7} \frac{3t^2 + 1}{t^2 - 5t + 2} = \frac{\lim_{t \rightarrow 7} 3t^2 + 1}{\lim_{t \rightarrow 7} t^2 - 5t + 2} =$ $\frac{3 \lim_{t \rightarrow 7} t^2 + \lim_{t \rightarrow 7} 1}{\lim_{t \rightarrow 7} t^2 - 5 \lim_{t \rightarrow 7} t + \lim_{t \rightarrow 7} 2} = \frac{3 \cdot 7^2 + 1}{7^2 - 5 \cdot 7 + 2}$ $= \frac{148}{16} = \frac{37}{4}$
7. $\lim_{u \rightarrow -2} \sqrt{9 - u^3 + 2u^2} =$ $\sqrt{\lim_{u \rightarrow -2} 9 - u^3 + 2u^2} =$ $\sqrt{\lim_{u \rightarrow -2} 9 - \lim_{u \rightarrow -2} u^3 + 2 \lim_{u \rightarrow -2} u^2} =$ $\sqrt{9 - (-2)^3 + 2 \cdot (-2)^2} = \sqrt{9 + 8 + 8} = 5$	8. $\lim_{x \rightarrow 3} \sqrt[3]{x + 5} (2x^2 - 3x) =$ $\left[\lim_{x \rightarrow 3} \sqrt[3]{x + 5} \right] \left[\lim_{x \rightarrow 3} (2x^2 - 3x) \right] =$ $\left[\sqrt[3]{\lim_{x \rightarrow 3} x + \lim_{x \rightarrow 3} 5} \right] \left[2 \lim_{x \rightarrow 3} x^2 - 3 \lim_{x \rightarrow 3} x \right] =$ $(\sqrt[3]{8})(9) = 18$
9. $\lim_{t \rightarrow -1} \left(\frac{2t^5 - t^4}{5t^2 + 4} \right)^3 =$ $\left(\frac{2 \lim_{t \rightarrow -1} t^5 - \lim_{t \rightarrow -1} t^4}{5 \lim_{t \rightarrow -1} t^2 + \lim_{t \rightarrow -1} 4} \right)^3 =$ $\left(\frac{2(-1^5) - (-1^4)}{5(-1^2) + 4} \right)^3 = \left(\frac{-3}{9} \right)^3 =$ $\left(-\frac{1}{3} \right)^3 = -\frac{1}{27}$	

Most of these require you to multiply an A+B factor by A-B or the reverse.

12. 5	13. 6
14. -3	15. DNE
16. DNE	17. 5/7
18. 11/10	19. 9/2
20. 1/3	21. -6
22. 6	23. 1/6
24. -4	25. -1/9
26. DNE	27. 1
28. 1	29. 1/128
30. 0	31. -1/2
32. -4/5	33. $2x^2$ this is the derivative of x^3
34. $-2x^{-3}$ this is the derivative of x^{-2}	

37. Note that

$$-1 \leq \cos(20\pi x) \leq 1 \text{ so we have } -x^2 \leq \cos(20\pi x) \leq x^2$$

Also note that $\lim_{x \rightarrow 0} -x^2 = 0$ and $\lim_{x \rightarrow 0} x^2 = 0$

So, by the squeeze theorem $\lim_{x \rightarrow 0} x^2 \cos(20\pi x) =$

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35. 4	36. 0
37. $\ln(x)$	38. 9
47. We can form an equation here $4c + 4 = 8 - 2c$ So we get $c = 2/3$	48. First note that $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$ Now we can get two equations in two unknowns $4a - 2b + 3 = 4$ $9a - 3b + 3 = 6 - a + b$ Solving these we get $a = \frac{1}{2}$ and $b = \frac{1}{2}$
51. a) this function can be written $\frac{(x - 1)(x + 1)(x^2 + 1)}{x - 1}$ So, the discontinuity at 1 can be removed. A function that agrees but is continuous at 1 is $f(x) = (x + 1)(x^2 + 1)$	51. b) this function can be written $\frac{x(x + 1)(x - 2)}{x - 2}$ So, the discontinuity at 2 can be removed. A function that agrees but is continuous at 2 is $f(x) = x(x + 1)$
51. c) The discontinuity at π is a jump discontinuity and cannot be removed.	

53.

The function is continuous.

Also $f(0) = 0$ and $10000 < f(100) < 10010$

So, by the intermediate value theorem there exists a number c , $0 < c < 100$ such that $f(c)=10000$.

55.

If $f(x) = -x^3 + 4x + 1$

then $f(-1) = -2$

and $f(0) = 1$

Since $-2 < 0 < 1$ by the intermediate value theorem there exists a number c on the interval $(-1,0)$ such that $f(c) = 0$

56.

Since $\ln(x) = x - \sqrt{x}$ we can form the function $f(x) = \ln(x) - x + \sqrt{x}$

then $f(2) = \sim .107$

and $f(3) = \sim -.159$

Since $-.159 < 0 < .107$ by the intermediate value theorem there exists a number c on the interval $(-1,0)$ such that $f(c) = 0$, which is a solution to our equation.

75.

Use an altitude scale such that the bottom of the mountain is at height=0 and the top is at height =1.

Let $H(t)$ be the monk's height on the first day at time t while ascending.

Let $G(t)$ be his height on the second day at time t while descending.

Let $D(t) = G(t)-H(t)$

Since H and G are continuous functions, D will be continuous.

At the start of the second day we have $D(7AM) = 1$.

At the end of the second day we have $D(7PM) = -1$.

So, by the intermediate value theorem there is a time t $7AM < t < 7PM$ such that $D(t) = 0$.
Clearly with $D(t)=0$ $H(t)=G(t)$.