Section 2.6 Example

$$\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{3x-5}=\lim_{x\to\infty}\frac{\sqrt{2+\frac{1}{x^2}}}{3-\frac{5}{x}}=\frac{\sqrt{2}}{3}$$

$$\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{3x-5}=\lim_{x\to-\infty}\frac{-\sqrt{2+\frac{1}{x^2}}}{3-\frac{5}{x}}=\frac{-\sqrt{2}}{3}$$

Why is this?

First note that we have a convention that for $x \geq 0$ $\sqrt{x} \geq 0$

Note that $\left(-\sqrt{x}\right)^2 = x$ so $-\sqrt{x}$ is also the square root of *x*.

Consider this equation.

$$x=\sqrt{x^2}$$

Is this true? Not if $x < 0$!

If $x<0$ then $x=-\sqrt{x^2}$

$$\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{3x-5}\cdot\frac{\frac{1}{x}}{\frac{1}{x}}=\lim_{x\to-\infty}\frac{\sqrt{2x^2+1}}{3x-5}\cdot\frac{\frac{-1}{\left(\sqrt{x}\right)^2}}{\frac{1}{x}}=$$

$$\lim_{x\to-\infty}\frac{\frac{-\sqrt{2x^2+1}}{\left(\sqrt{x}\right)^2}}{3-\frac{5}{x}}=\lim_{x\to-\infty}\frac{-\sqrt{2+\frac{1}{x^2}}}{3-\frac{5}{x}}=-\frac{\sqrt{2}}{3}$$