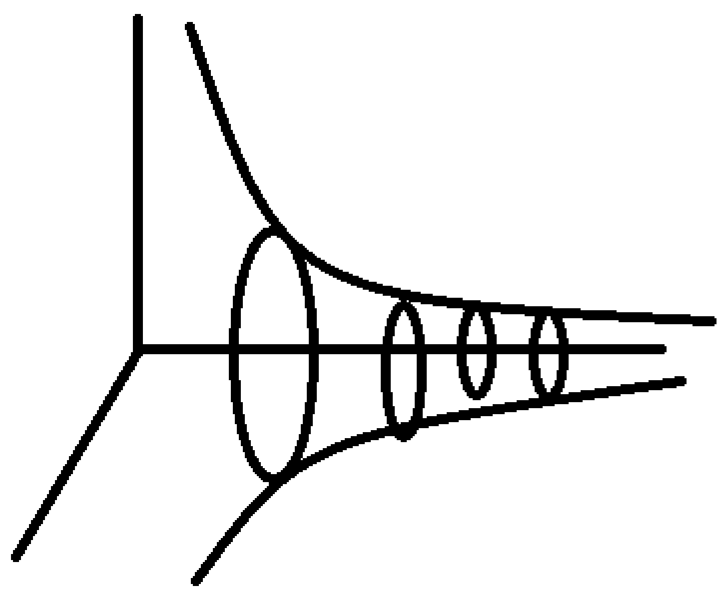
Math 109 Calc 1 Lecture 36

Volume vs. Surface Area

Consider taking the function $f(x) = \frac{1}{x}$ on the interval $[1, \infty)$

Now create a volume by spinning the function around the *x*-axis.



First let’s investigate the volume inside this object.

At each point we have a thin disk of volume $\pi r^2 dx$ where $r = \frac{1}{x}$

So, we can write the integral as follows:

$$V = \lim_{a\to\infty} \int_1^a \pi \frac{1}{x^2}\, dx = \pi \left[\lim_{a\to\infty} \int_1^a \frac{1}{x^2}\, dx \right] = \pi \left[\lim_{a\to\infty} \left(-\frac{1}{x} \right) \right]_1^a =$$
$$\pi \left[\lim_{a\to\infty} \left(-\frac{1}{a} - \frac{-1}{1} \right) \right] = \pi \left[\lim_{a\to\infty} \left(1 - \frac{1}{a} \right) \right]$$

Now evaluating the limit, you can see that we are left with $V = \pi$

Next let’s investigate the surface area of this object.

Each small ring has a circumference of $2\pi r$ and a volume of $2\pi r\, dx$

So, the surface area is

$$S = \lim_{a\to\infty} \int_1^a 2\pi \frac{1}{x}\, dx = 2\pi \left[\lim_{a\to\infty} \int_1^a \frac{1}{x}\, dx \right] = 2\pi \left[\lim_{a\to\infty} \left(\ln(x) \right) \right]_1^a =$$
$$2\pi \left[\lim_{a\to\infty} \left(\ln(a) - \ln(1) \right) \right] = 2\pi \left[\lim_{a\to\infty} \left(\ln(a) \right) \right]$$

But the function $f(x) = \ln(x)$ grows without bound, so we are left with the perplexing results that $S = \infty$.

Considering this from a physical world perspective is confusing.

Imagine that we tried to fill up this object with paint.
It would take a small finite volume of paint to fill it.

But if we were to try to paint the outside of it, it would take an infinite amount of paint.