Math 109 Calc 1 Lecture 11
Review of Trigonometry

You must know the following definitions

Tangent $tan(x) = \frac{sin(x)}{cos(x)}$

Co-Tangent $ctn(x) = cot(x) = \frac{1}{tan(x)} = \frac{cos(x)}{sin(x)}$

Secant $sec(x) = \frac{1}{cos(x)}$

Cosecant $csc(x) = \frac{1}{sin(x)}$

Note the asymmetric definitions secant = 1/co-sine, co-secant = 1/sine
You should know what these functions look like when graphed, and you should know their domains and ranges.

Important Identities

Some of these you will need to memorize. Some can be determined from each other.
The most important Identity: The Pythagorean Identity

$$sin^2(x) + cos^2(x) = 1$$

Helpful at times are the odd/even function identities

$sin(-x) = -sin(x)$ (Odd)
$cos(-x) = cos(x)$ (Even)
$tan(-x) = -tan(x)$ (Odd)

You should also recognize the common versions of this you might come across
$sin^2(x) = 1 - cos^2(x)$ or $cos^2(x) = 1 - sin^2(x)$

And the partial versions
$sin(x) = \pm\sqrt{1 - cos^2(x)}$ and $cos(x) = \pm\sqrt{1 - sin^2(x)}$

The Sum and Difference formulas

$$sin(A + B) = sin(A)cos(B) + cos(A)sin(B)$$
$$sin(A - B) = sin(A)cos(B) - cos(A)sin(B)$$

$cos(A + B) = cos$(A)cos(B)-sin(A)sin(B)
$cos(A - B) = cos$(A)cos(B)+sin(A)sin(B)

You can learn how to derive the tangent formula or memorize it.

$$tan(A \pm B) = \frac{tan(A) \pm tan(B)}{1 \mp tan(A)tan(B)}$$

From these identities you can find the double angle formulas

$sin(2A) = 2sin(A)cos(A)$

$cos(2A) = cos^2(A) - sin^2(A) = 2cos^2(A) - 1 = 1 - 2sin^2(A)$

$tan(2A) = \dfrac{2tan(A))}{1 - tan^2(A)}$

Using the double angle identities, it is possible to derive the half angle identities. These are less commonly used.

$$sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - cos(x)}{2}}$$

$$cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + cos(x)}{2}}$$

$$sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - cos(x)}{1 + cos(x)}} = \frac{sin(x)}{1 + cos(x)} = \frac{1 - cos(x)}{sin(x)}$$

To make use of these, and in general you should know the sign of the trig functions in the different quadrants.

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<td>sine and cosecant
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<td>cosine and secant
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<td>tangent and co-tangent
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