Math 109 Calc 1 Lecture 1

Introduction

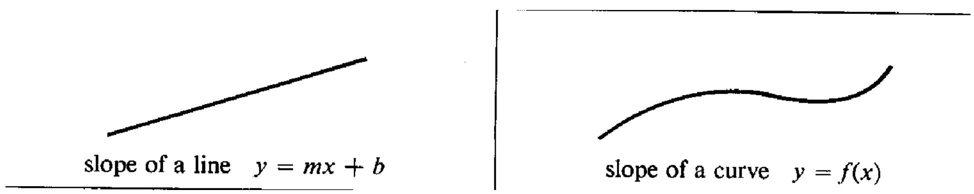
There are two main subjects you will learn in Calculus,

1. Differentiation
2. Integration

Differentiation is about finding slopes and integration is about finding the area under curves.

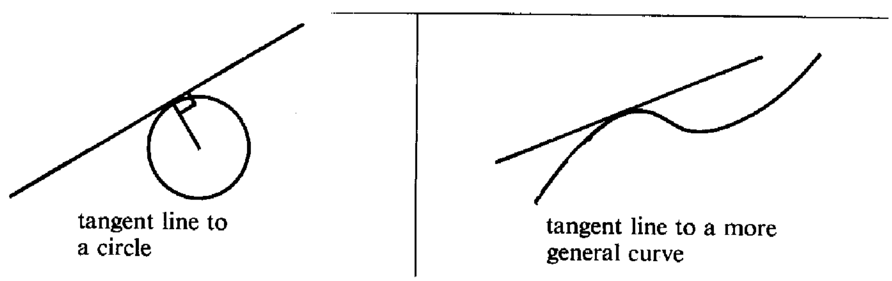
In your previous math courses, you've learned the slope of a line

In this course we will learn about the slopes of more general functions.



In your previous math courses, you've learned about the tangent to a circle.

In this course you will learn about a tangent to a more general curve

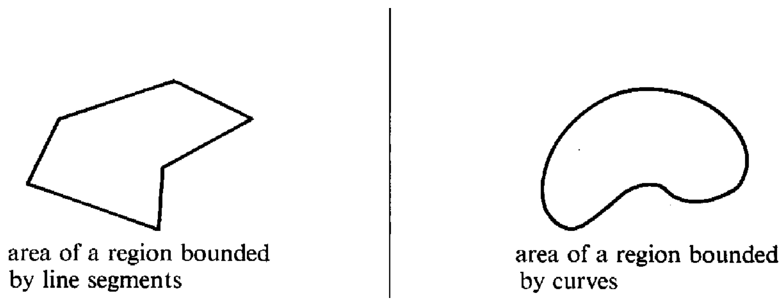


In previous math classes you may have learned about motion where velocity and acceleration are constant.

In this class you will learn about motion with varying velocity and varying acceleration

In previous math classes you learned to calculate the area in a polygon.

In this class you will learn to calculate the area of a region bounded by curves.

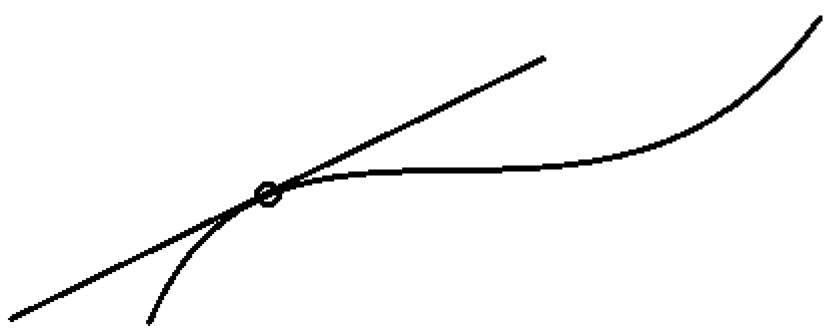


In previous math classes you learned about angle measurements in degrees and radians.

In this class you will see why radian measure makes a lot of sense.

Differentiation

The slope at a point on a function is also the slope of a tangent to the function at that same point.



A function whose values are the slopes of another function is called the <u>derivative</u> of that function.

If we have a function *f*(x), there are different ways we indicate the derivative of that function. One way is with an apostrophe

$$f\,'(x)$$

This is known as Newtonian notation. Sir Isaac Newton was one of the two mathematicians who is credited with inventing calculus.

Another way is

$\frac{d}{dx}f(x) =$

This is known as Leibniz notation after Gottfried Wilhelm Leibniz.

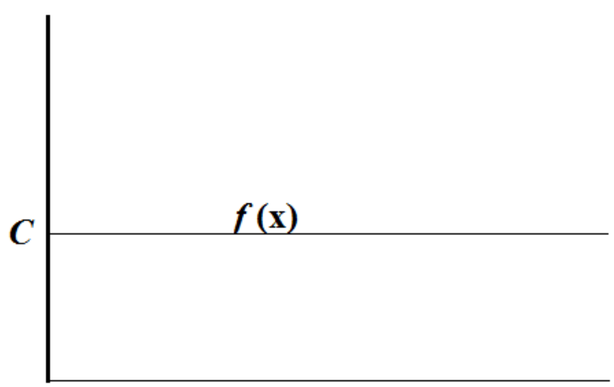
If we are describing a function by a formula y = …, we can write this as

$\frac{dy}{dx} =$

This is notation. Note this does not connote a fraction. Sometimes the *dy* and the *dx* are known as <u>infinitesimals</u>. It's fine to think of them as values that are both infinitesimally small, but do not put too much weight on this idea.

Here are some very simple functions and their derivatives.

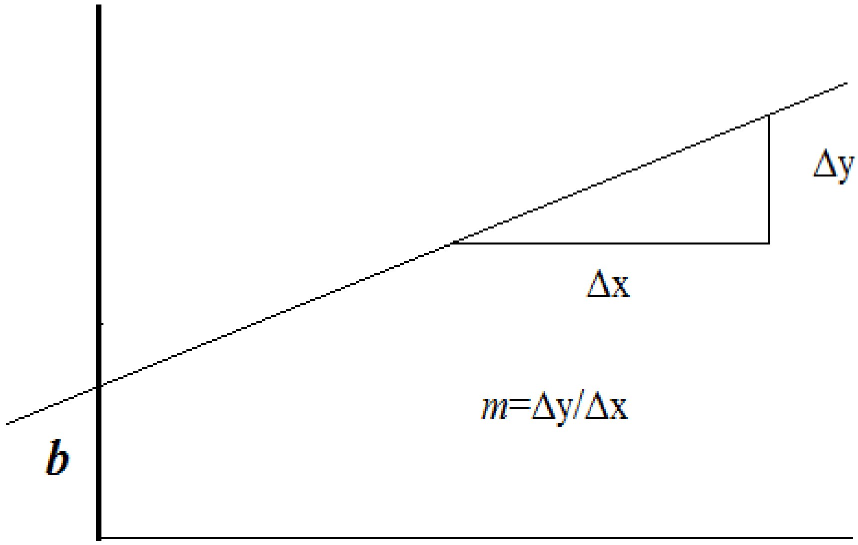
Let $f(x) = C$ where *C* is just some constant.



At every point of *f* (x) the slope is? Zero. So, the derivative function is

$f\,'(x) = 0$

If instead our function is $f(x) = mx + b$



Our derivative function is a constant function

$$f'(x) = \frac{d}{dx}(mx + b) = m$$

Notice that the constant b has no effect on the derivative function.
That makes sense since you can translate the function up and down but the slope does not change.

There's a hint here of what is to come.
Since we already know that for
$f(x) = b, that\ f\ '(x) = \frac{d}{dx}b = 0$, this suggests the following

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

This could be very helpful as it means we can differentiate terms separately.

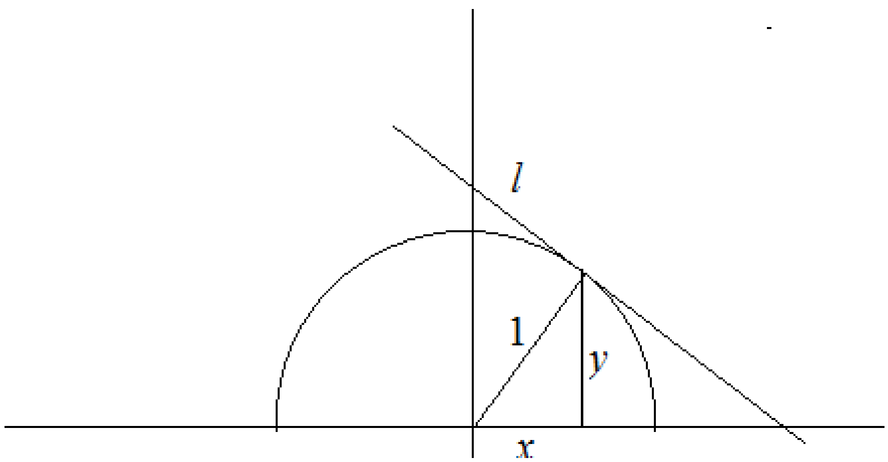
We will find out later that this is the case.

Let's try to find one more derivative.
Consider the function $f(x) = \sqrt{1 - x^2}$

Well what is this? If we set y=f (x) we see

$$y = \sqrt{1 - x^2}$$
$$y^2 = 1 - x^2$$
$$x^2 + y^2 = 1$$

Which is the equation of a circle. So we want to find the derivative of the top half of a circle.



Note: $y = \sqrt{1 - x^2}$ (by the Pythagorean theorem)

The slope of the radius is $\frac{\Delta y}{\Delta x} = \frac{y}{x} = \frac{\sqrt{1-x^2}}{x}$

So the perpendicular slope of line l is the negative reciprocal,

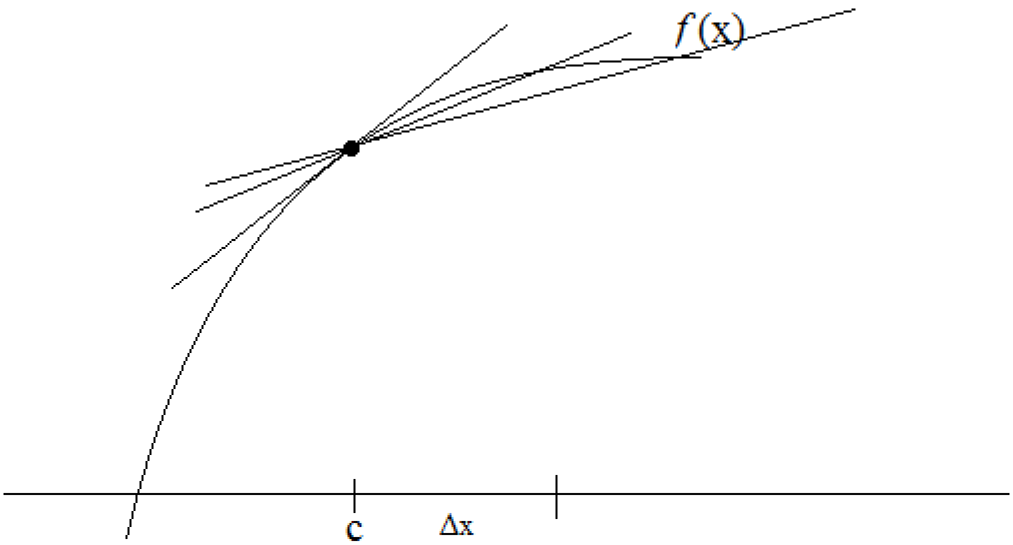
$$f'(x) = \frac{-x}{\sqrt{1 - x^2}}$$

Think about this for a minute.

What happens at x=0?

What happens at x=1 and x=-1?

For a more general derivative we look at successive approximations of the slope.



By reducing Δx we get a line closer and closer to our tangent, with slope closer and closer to the value of our derivative at the point *c*.

The formula for this approximation is

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(x + \Delta x)}{\Delta x}$$

To find the derivative we want to reduce Δ*x* to zero, but that would give us a value $\frac{0}{0}$.

At this point we need to introduce the idea of a limit.

Once we've built up some more foundation, we will return to this formula.