Math 109 Calc 1

Lecture 0-1

Review (Recap)

You should know what the **root** or **zero** of a polynomial equation is, a value that is a solution to the equation.

Example: $x^2 - 2x + 1 = 0$ has a root of x = 1

You should know how to do long division and possibly synthetic division

Example: $\frac{x^3 - 6x^2 + 11x - 6}{x - 1}$ (long division or synthetic division $\frac{x^3 - 6x^2 + 11x - 5}{x^2 - x - 1}$ (long division only Given the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

You should know that the **discriminant** $b^2 - 4ac$ can tell you how many real and/or complex roots the equation has.

Example: $x^2 + x + 1 = 0$ $b^2 - 4ac = 1^2 - 4(1)(1) = -3$ which is negative, so the equation has two complex roots.

You should know what the inverse function of a function is and how to find one.

Example: $f(x) = x^2 + 5$

The graphs of a function and an inverse function will be mirror images across the line y = x

Example: $f(x) = x^2$, $f^{-1}(x) = \sqrt{x}$



Pre-Calculus

You should know what an **increasing function** or **decreasing function** is.

An increasing function is one where if a < b then for all a and b in the domain of the function.

An increasing function is one where if a < b then $f(a) \ge f(b)$ for all a and b in the domain of the function.

You should know what a **rational function** is. A rational function is one in the form $f(x) = \frac{P(x)}{Q(x)}$ where *P* is a polynomial function and *Q* is a polynomial function of degree 1 or greater. Example: $f(x) = \frac{x^2 + 1}{x - 2}$ The domain of a rational function if not specified is assumed to be all real numbers where $Q(x) \neq 0$. You should know how to find the **vertical asymptotes** of a rational function, the **horizontal asymptotes** if there are any, and the end behavior. Example: $f(x) = \frac{1}{x}$ has a vertical asymptote at x = 0 and a horizontal asymptote at y = 0.

Example:
$$f(x) = \frac{x^3 + 5}{x^2 + 2}$$

This function has no asymptotes. Its end behavior is to increase as x gets large and to decrease as x gets large in the negative direction.

You should know what an **exponential function** is Example: $f(x) = 10^x$

You should about log functions, the inverse functions of exponential functions. Example: $f(x) = \log_{10} x$

You should know the laws of exponents

$$a^{n} \cdot a^{m} = a^{n+m}$$
$$(a^{n})^{m} = a^{nm}$$
$$\frac{a^{n}}{a^{m}} = a^{n-m}$$
$$\sqrt{a} = a^{\frac{1}{2}}$$
$$\sqrt{a} = a^{\frac{1}{n}}$$

and the laws of logs. $\log ab = \log a + \log b$

$$\log \frac{a}{b} = \log a - \log b$$
$$\log a^{n} = n \log a$$

You should know what a periodic function is, f(x+p) = f(x)

You should know the **period** of these trigonometric functions.

Period 2π : sin(), cos(), sec(), csc()

Period π : tan(), ctn()

You should know what the graphs of these functions look like.

You should be able to find the exact values of some specific values without a calculator.

$$\left\{0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi...\right\}$$

Example:
$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Conic Sections (Optional)

It may be useful to know the equations of the following conic sections

A circle centered at (a,b) with radius r $(x-a)^{2} + (y-b)^{2} = r^{2}$

A parabola with vertex at (a,b)

$$(y-b) = k(x-a)^2$$
 - Vertical
 $(x-b) = k(y-a)^2$ - Horizontal

An ellipse with center at (a,b) and A and B the semi-major and semi-minor axes

$$\frac{(x-a)^2}{A^2} + \frac{(y-b)^2}{B^2} = 1$$

A hyperbola with center at (a,b)

$$\frac{(x-a)^2}{A^2} - \frac{(y-b)^2}{B^2} = 1$$