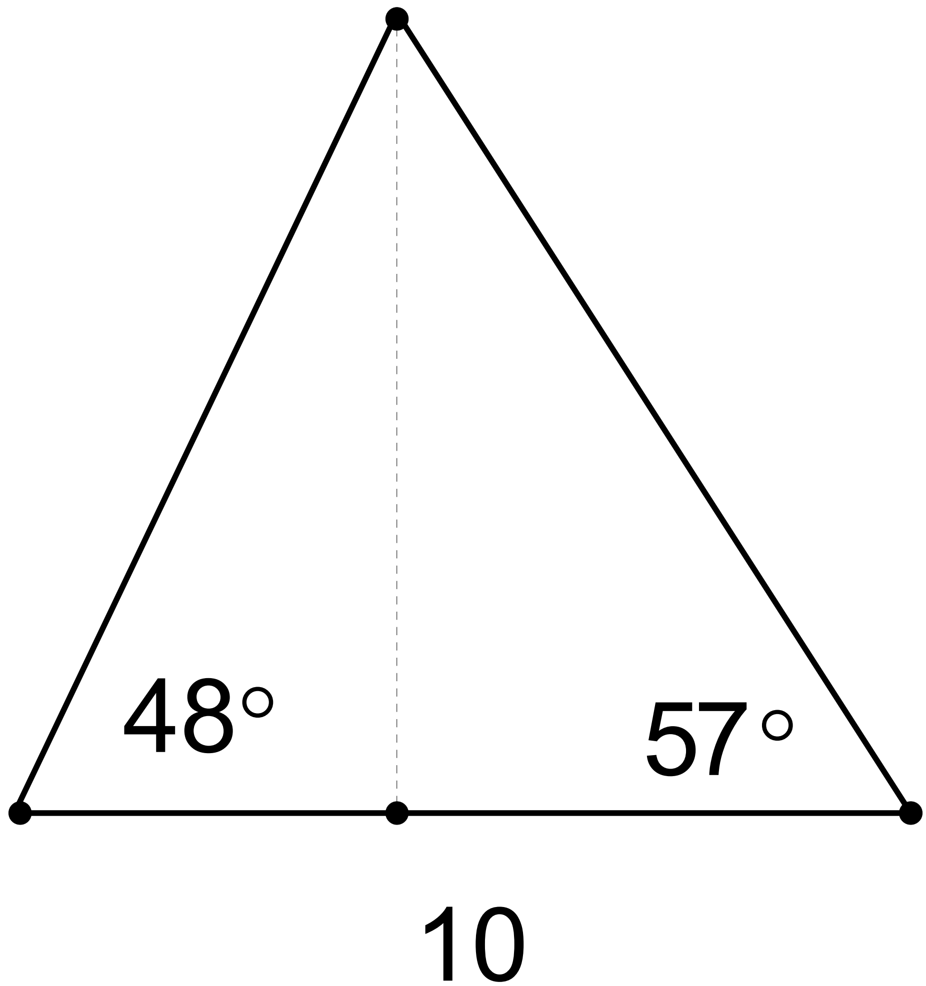
Law of Sines

What about this situation?   ASA

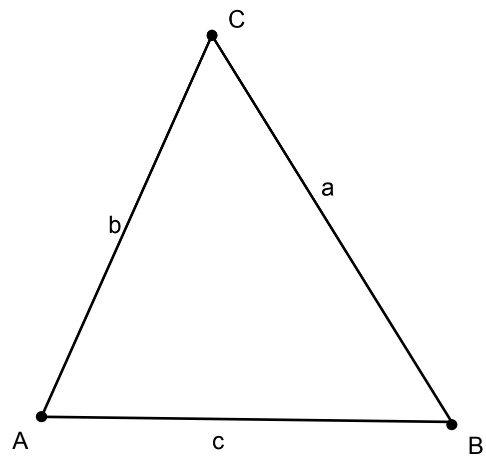


Note that dropping an altitude doesn't help much.

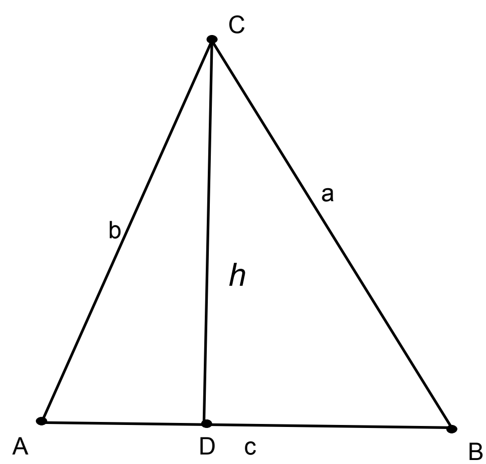
We need a new theorem to solve this: The Law of Sines

Show DVD Law of Sines

First we show derive the law of Sines!   Take a general acute triangle.



Drop an altitude of length h



Now Note the following:

$\frac{h}{b} = \sin A$ $\frac{h}{a} = \sin B$

multiplying by b and a respectively:

$h = b \sin A$ $h = a \sin B$

By transitivity we have

$b \sin A = a \sin B$ or after dividing both sides by ab you get

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Repeat this all for an altitude dropped from A or B and you get the complete Law of Sines

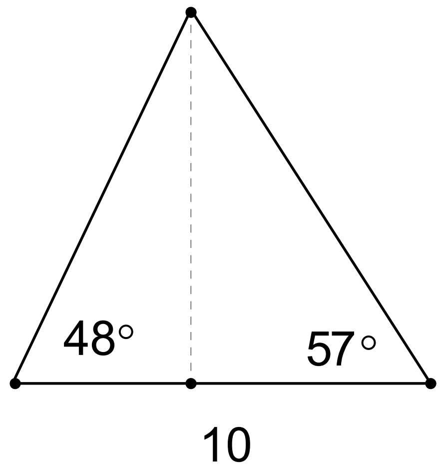
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

THIS IS THE LAW OF SINES

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

So given any two angles and a corresponding side, or any two sides and a corresponding angle, the missing angle or side can be calculated!

How can we use these? Let's go back to the previous two unsolved problems
ASA



We can easily find that the third angle is 180° - 105° = 75°.

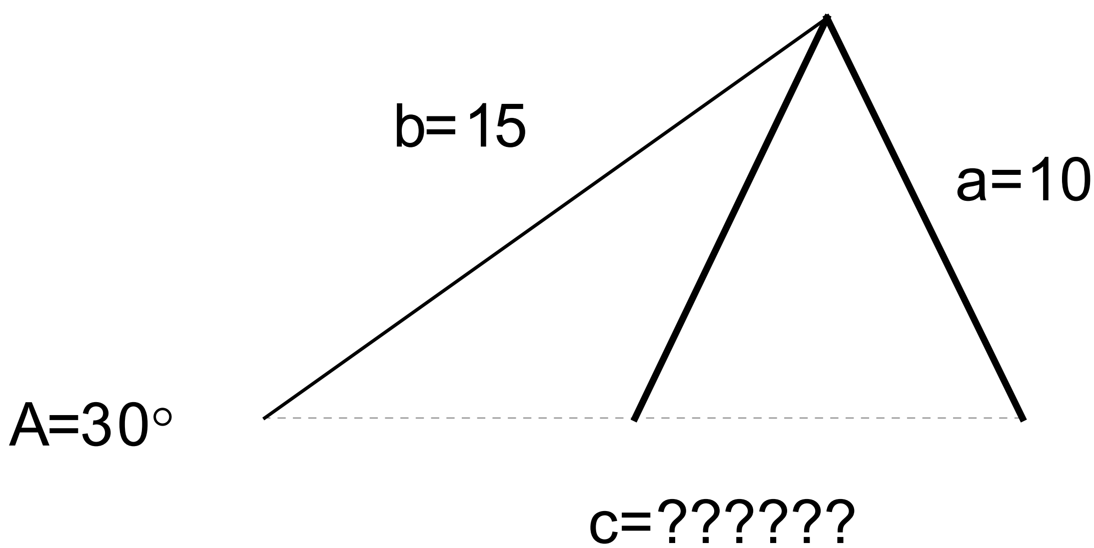
Using the Law of Sines we have

$$\frac{\sin\left(75^\circ\right)}{10}=\frac{\sin\left(57^\circ\right)}{b}=\frac{\sin\left(48^\circ\right)}{a}$$

We can solve for a or b by cross multiplying and dividing!
$a = 7.69$ $b = 8.68$

An interesting problem, SSA

There is a problem with SSA, the solution may not be unique:



Here we have $\dfrac{\sin\left(30^\circ\right)}{10}=\dfrac{\sin\left(B\right)}{15}$

Solving we find that $\sin\left(B\right)=\dfrac{10\sin\left(30^\circ\right)}{15}=\dfrac{1}{3}$

The principle angle is therefore $B=\sin^{-1}\left(\dfrac{1}{3}\right)\approx 19.5^\circ$

But B can be either acute or obtuse.

For the inverse values in the 2nd quadrant we get the complementary angle $180^\circ-19.5^\circ=160.5^\circ$

HW: 6.5:3, 4, 5, 10, 13, 14, 19, 20