

Working with Trig Functions

The exact values of the sine and cosine can be determined exactly for angles that multiples of 30° and 45° .

Functions: We can find an exact expression for any multiple of 30° or 45°

Angle	Sine	Cos	Angle	Sine	Cos	Angle	Sine	Cosine	Angle	Sine	Cos
0°			90°			180°			360°		
30°			120°			210°			300°		
45°			135°			225°			315°		
60°			150°			240°			330°		

Solutions

Angle	(cos,sin)	Angle	(cos,sin)	Angle	(cos,sin)	Angle	(cos,sin)
0°	$(+1,0)$	90°	$(0,+1)$	180°	$(-1,0)$	360°	$(0,-1)$
30°	$\left(+\frac{\sqrt{3}}{2}, +\frac{1}{2}\right)$	120°	$\left(-\frac{1}{2}, +\frac{\sqrt{3}}{2}\right)$	210°	$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$	300°	$\left(+\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
45°	$\left(+\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right)$	135°	$\left(-\frac{1}{\sqrt{2}}, +\frac{1}{\sqrt{2}}\right)$	225°	$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$	315°	$\left(+\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$
60°	$\left(+\frac{1}{2}, +\frac{\sqrt{3}}{2}\right)$	150°	$\left(-\frac{\sqrt{3}}{2}, +\frac{1}{2}\right)$	240°	$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$	330°	$\left(+\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Special Angles with Exact Values

Using our knowledge of special triangles from geometry:

30/60/90 triangles:

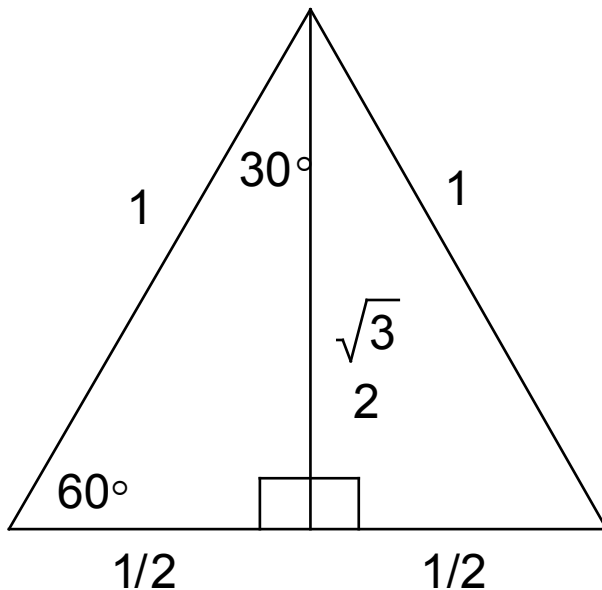
Take an equilateral triangle with sides 1 whose angles must all be 60° .

Drop a perpendicular from it's highest point to the base.

This divides the triangle into two congruent triangles.

By symmetry the angles of each of these triangles must be 30/60/90 degrees.

The base is $1/2$ and the hypotenuse is 1 so by the Pythagorean theorem we get the second leg to be $\frac{\sqrt{3}}{2}$



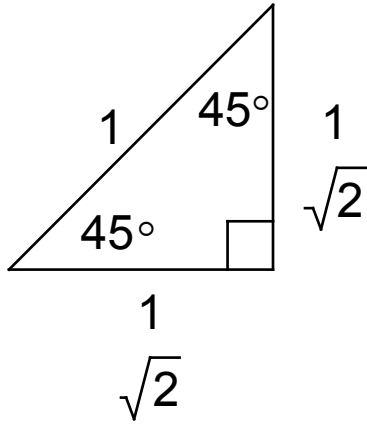
This tells us that

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

$$\cos(60^\circ) = \frac{1}{2}$$

Isosceles right triangles:

Given a right isosceles triangle with hypotenuse 1 we know immediately that the smaller angles are 45° and by the Pythagorean theorem, the legs are $\frac{1}{\sqrt{2}}$



This tells us that

$$\sin(45^\circ) = \frac{1}{\sqrt{2}}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

Definitions of the other Trigonometric Functions

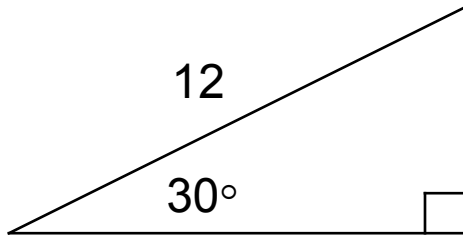
Tangent: $\tan(x)$ $\frac{\sin(x)}{\cos(x)}$

Cotangent: $\cot(x)$ or $ctn(x)$ $\frac{\cos(x)}{\sin(x)}$

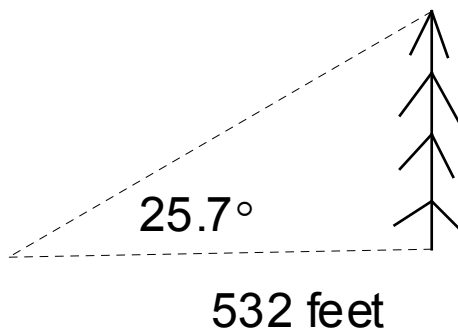
Secant: $\sec(x)$ $\frac{1}{\cos(x)}$

Cosecant: $\csc(x)$ $\frac{1}{\sin(x)}$

Solving Triangle Problems



Tree Problem



How tall is the tree?

HW: 5.2: 5-10, 13-15, 23-26

HW: 6.2: 3, 4, 9, 10, 15-20, 29-32, 39, 40,

HW: 6.3 5-8, 37, 38, 47, 48, 49