Trigonometry 14 Mathematics 108

Mention Study Guide, Cheat Sheet and Practice Exam on the web

Solving Equations with Trigonometric Functions

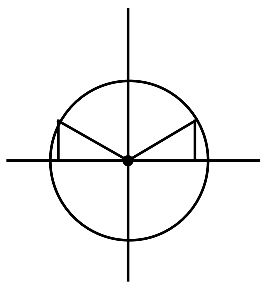
When solving equations with trig functions, there are a few techniques.
Keep in mind that because trig functions are periodic, there might be periodic solutions.
These solutions may have a different period from the standard trig functions.

A simple example

$$\sin\theta = \frac{1}{2} \rightarrow \theta = \sin^{-1}\frac{1}{2}$$

Note this is the same strategy we always use, to get the unknown variable alone.

In the first period of the sine function we know this value occurs twice.



$$\theta = \frac{\pi}{6} \text{ and } \theta = \frac{5\pi}{6}$$

But since sine is 2π periodic, the full solution is

$$\theta = \frac{\pi}{6} + 2\pi n \text{ and } \theta = \frac{\pi}{6} + 2\pi n$$

A calculator example

$$\cos\theta = .65 \rightarrow \theta = \cos^{-1}.65$$

We solve this with a calculator getting $\theta = .863$
But there will be a second solution in the first 2π
-.863 or 2π -.863= 5.42

So the solution is

$$\theta = .863 + 2\pi n \text{ and } \theta = 5.42 + 2\pi n$$

Factoring

If you can factor the equation so that two or more factors equal zero, you can set each of them equal to zero.

Example

$5\sin\theta\cos\theta + 4\cos\theta = 0$

$\cos\theta(5\sin\theta - 4) = 0$

$\cos\theta = 0 \text{ or } \sin\theta = \frac{4}{5}$

$\theta = \cos^{-1} 0 \text{ or } \theta = \sin^{-1}\frac{4}{5} \approx .927$

The first gives solutions

$\theta = \pi n$

The second gives solutions

$\theta = .927 \text{ and } \theta = \pi - .927 \approx 2.21$

So finally we get

$\theta = \pi n \text{ or } \theta = .927 + 2\pi n \text{ or } \theta = 2.21 + 2\pi n$

Note that we could keep this exact by leaving $\sin^{-1}$ unresolved

$\theta = \pi n \text{ or } \theta = \sin^{-1}\frac{4}{5} + 2\pi n \text{ or } \theta = \left(\pi - \sin^{-1}\frac{4}{5}\right) + 2\pi n$

Factoring a Quadratic

$$2\cos^2\theta - 7\cos\theta + 3 = 0$$

In this example, treat cos as a separate variable. You can factor or use the quadratic equation:

$$2x^2 - 7x + 3 = 0$$
$$(2x-1)(x-3) = 0$$
$$x = \frac{1}{2}, 3$$

This becomes

$$\theta = \cos^{-1}\frac{1}{2} \text{ or } \theta = \cos^{-1} 3$$

The latter is undefined because 3 is not in the domain of $\cos^{-1}$ so we end up with solutions

$$\theta = \frac{\pi}{3} + 2\pi n \text{ and } \theta = \frac{2\pi}{3} + 2\pi n$$

This same strategy will work even if you need to use the quadratic equation

$$9\cos^2\theta - 6\cos\theta - 1 = 0$$
$$\cos\theta = \frac{6 \pm \sqrt{36+36}}{18} = \frac{1 \pm \sqrt{2}}{3}$$

This will produce 4 θ values in the first 2π

An example where squaring is necessary

$$\cos\theta + 1 = \sin\theta$$

Here we have two different trig functions, but we need to convert one using the Pythagorean identity $\sin\theta = \sqrt{1-\cos^2\theta}$

$$\cos\theta + 1 = \sqrt{1-\cos^2\theta}$$

But now we will have to square both sides

$$\cos^2\theta + 2\cos\theta + 1 = 1 - \cos^2\theta$$
$$2\cos^2\theta + 2\cos\theta = 0$$
$$\cos\theta\left(\cos\theta + 1\right) = 0$$
$$\cos\theta = 0, -1$$

This gives $\theta = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

With periodicity

$\theta = \frac{\pi}{2} + \pi n$ and $\theta = \pi + 2\pi n$

An example where the period is not as obvious

$2\sin 3\theta - 1 = 0$

In this example we solve for $\sin 3\theta = \frac{1}{2}$

Applying the inverse sine function we know that

$3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

It is tempting to divide by 3 at this point to get solutions

$\theta = \frac{\pi}{18}, \frac{5\pi}{18}$

But it is better to first account for periodicity

$3\theta = \frac{\pi}{6} + 2\pi n$ and $3\theta = \frac{5\pi}{6} + 2\pi n$

and then divide giving

$\theta = \frac{\pi}{18} + \frac{2\pi n}{3}$ and $3\theta = \frac{5\pi}{18} + \frac{2\pi n}{3}$

The alternative is take $\theta = \frac{\pi}{18}, \frac{5\pi}{18}$ but realize the period of $\sin 3\theta$ is $\frac{2\pi}{3}$

HW 7.4: 17, 18, 21, 22, 25, 33, 41, 42
7.5: 4, 10, 17, 18