Trigonometry 13 Mathematics 108

Please Comment about a cheat sheet for the final.

More Formulas can be created

Example

$$\sin 3x = \sin 2x + x = \sin 2x \cos x + \cos 2x \sin x =$$
$$2\sin x \cos x \cos x + \left(\cos^2 x - \sin^2 x\right)\sin x =$$
$$\sin x\left(2\cos^2 x + \cos^2 x - \sin^2 x\right) =$$
$$\sin x\left(3\cos^2 x - \sin^2 x\right) =$$
$$\sin x\left(3\left(1-\sin^2 x\right) - \sin^2 x\right) =$$
$$\sin x\left(3 - 4\sin^2 x\right) =$$
$$2\sin x - 4\sin^2 x$$

Similarly we can find

$\cos 3x \quad \sin 4x \quad \cos 4x \quad \tan 3x \quad \tan 4x$

Half Angle Formulas

Starting with

$$\cos(2x) = 2\cos^2(x) - 1$$

we substitute getting

$$\cos(x) = 2\cos^2\left(\frac{x}{2}\right) - 1$$

Solving for $\cos\left(\frac{x}{2}\right)$

We get

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}}$$

Starting with

$$\cos(2x) = 1 - 2\sin^2(x)$$

we substitute getting

$$\cos(x) = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

Solving for $\sin\left(\frac{x}{2}\right)$

We get

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}$$

Finally, there are three versions of the tangent half angle

$$\tan\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

$$\tan\left(\frac{x}{2}\right)=\frac{1-\cos(x)}{\sin(x)}$$

$$\tan\left(\frac{x}{2}\right)=\frac{\sin(x)}{1+\cos(x)}$$

Example of half angle formula:

$$\sin\left(15^\circ\right)=\sin\left(\frac{30^\circ}{2}\right)=\pm\sqrt{\frac{1-\cos\left(30^\circ\right)}{2}}=$$

$$\pm\sqrt{\frac{1-\sqrt{3}/2}{2}}=\pm.258819$$

So which is it? .2588 or -.2588

Well 15° is in the first quadrant so .2588!

Product-Sum

We'd like to find formulas of the form

$\sin x \cos y =$

We start by adding

$\sin(x+y)+\sin(x-y)$

We get

$$\sin(x+y)+\sin(x-y)=\sin x\cos y+\cos x\sin y+six\cos y-\cos x\sin y=$$
$$2\sin x\cos y$$

Dividing by two we get

$$\sin x\cos y=\frac{\sin(x+y)+\sin(x-y)}{2}$$

If instead we try

$\sin(x+y)+\sin(x-y)$

We get

$$\sin(x+y)-\sin(x-y)=\sin x\cos y+\cos x\sin y-six\cos y+\cos x\sin y=$$
$$2\cos x\sin y$$

again

$$\cos x\sin y=\frac{\sin(x+y)-\sin(x-y)}{2}$$

Repeating this using the cosine sum and difference formulas

$$\cos(x+y)+\cos(x-y)=$$
$$\cos x\cos y-\sin x\sin y+\cos x\cos y+\sin x\sin y=$$
$$2\cos x\cos y$$

Dividing by 2 we get

$$\cos x\cos y=\frac{\cos(x+y)+\cos(x-y)}{2}$$

And finally

$$\cos(x+y)-\cos(x-y)=$$
$$\cos x\cos y-\sin x\sin y-\cos x\cos y-\sin x\sin y=$$
$$-2\sin x\sin y$$

Dividing by -2 we get

$$\sin x\sin y=\frac{\cos(x-y)-\cos(x+y)}{2}$$

Summary

$$\sin x \cos y = \frac{\sin(x+y)+\sin(x-y)}{2}$$
$$\cos x \sin y = \frac{\sin(x+y)-\sin(x-y)}{2}$$
$$\cos x \cos y = \frac{\cos(x+y)+\cos(x-y)}{2}$$
$$\sin x \sin y = \frac{\cos(x-y)-\cos(x+y)}{2}$$

We can use these to find sum formulas

Let $A = x + y \quad and \quad B = x - y$

so $x = \dfrac{A+B}{2} \quad y = \dfrac{A-B}{2}$

$$\sin x \cos y = \frac{\sin(x+y)+\sin(x-y)}{2}$$

becomes

$$\sin\frac{A+B}{2}\cos\frac{A-B}{2} = \frac{\sin(A)+\sin(A)}{2}$$

We multiply by 2 and rename the varialbes

$$\sin(x)+\sin(y) = 2\left[\sin\frac{x+y}{2}\cos\frac{x-y}{2}\right]$$

We repeat this using the other three product formulas and we get

$$\sin(x)+\sin(y) = 2\left[\sin\frac{x+y}{2}\cos\frac{x-y}{2}\right]$$
$$\sin(x)-\sin(y) = 2\left[\sin\frac{x+y}{2}\cos\frac{x-y}{2}\right]$$
$$\cos(x)+\cos(y) = 2\left[\cos\frac{x+y}{2}\cos\frac{x-y}{2}\right]$$
$$\cos(x)-\cos(y) = -2\left[\sin\frac{x+y}{2}\sin\frac{x-y}{2}\right]$$

Here's something you can do with these formulas:

Express $\cos(3x)+\cos(7x)$ as a product.

$$\cos(3x)+\cos(7x)=2\cos\left(\frac{3x+7x}{2}\right)\cos\left(\frac{3x-7x}{2}\right)=$$
$$2\cos 5x\cos(-2x)=2\cos 5x\cos 2x$$

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