USF Math 108 Final Sample Problems Solutions

Simplify

1. $\dfrac{\left(3xy^2z\right)^2}{yz^{-2}} = \dfrac{3^2x^2y^4z^2}{y^2z^{-4}} = 9x^2y^2z^6$

Simplify but leave as an exact answer

2. $\dfrac{\sqrt{5^2\cdot 6^3}}{\sqrt{3^2\cdot 5}} = \sqrt{\dfrac{5\cdot5\cdot2\cdot3\cdot2\cdot3\cdot2\cdot3}{3\cdot3\cdot5}} = \sqrt{5\cdot2\cdot2\cdot2\cdot3} = 2\sqrt{5\cdot2\cdot3} = 2\sqrt{30}$

3. Solve this inequality $|3x+4|<7$

For $3x+4\ge 0$ $\quad 3x+4<7 \to 3x<3 \to x<1$

For $3x+4<0$ $\quad -\left(3x+4\right)<7 \to 3x+4>-7 \to 3x>-11 \to x>\dfrac{-11}{7}$

So the solution is $\dfrac{-11}{7}<x<1$

4. What are the roots of this function $f\left(x\right)=x^4-5x^2+2$

$$y=x^2$$
$$y^2-5y+2=0$$
$$y=\frac{5\pm\sqrt{25-8}}{2}=\frac{5\pm\sqrt{17}}{2}$$
$$x=\pm\sqrt{\frac{5\pm\sqrt{17}}{2}}$$

5. Divide using long or synthetic division $\dfrac{x^5-x^4+x^3+4}{x-2}$

2	1	−1	1	0	0	4
		2	2	6	12	24
	1	1	3	6	12	28

$$\frac{x^5-x^4+x^3+4}{x-2}=x^4+x^3+3x^2+6x+12+\frac{28}{x-2}$$

6. Are the following functions even, odd, both or neither

a) $f(x) = 3x^2 + 1$

$f(-x) = 3(-x)^2 + 1 = 3x^2 + 1 = f(x)$ Even

b) $f(x) = x\cos x$

$f(-x) = (-x)\cos(-x) = -x\cos(x) = -f(x)$ Odd

7. What is the remainder when $x^{16} - 2x^{15} + 5$ **is divided by** $x - 2$

Using the remainder theorem

$2^{16} - 2 \cdot 2^{15} + 5 = 2^{16} - 2^{16} + 5 = 5$

8. What are all the possible rational roots of $f(x) = 5x^4 - 3x^3 + 2x^2 - x + 1$

Possible rational roots are 1 and -1

9. Find all the roots (real or complex) of $f(x) = x^5 - x^4 + 5x^3 - 5x^2$

$x^5 - x^4 + 5x^3 - 5x^2 = x^2\left(x^3 - x^2 + 5x - 5\right) =$

$x^2\left(\left(x^3 - x^2\right) + (5x - 5)\right) = x^2\left(x^2(x-1) + 5(x-1)\right) = x^2\left(x^2 + 5\right)(x-1)$

The 5 roots are 0 with multiplicity 2, 1, and $\pm\sqrt{5}i$

10. Simplify: $\log_4 1$

$\log_4 1 = x \to 4^x = 1 \to x = 0$

11. Simplify: $\log_5 \frac{1}{125}$

$\log_5 \frac{1}{125} = \log_5 5^{-3} = -3\log_5 5 = -3$

12. Simplify: $\log_6 8 + \log_6 9 - \log_6 2$

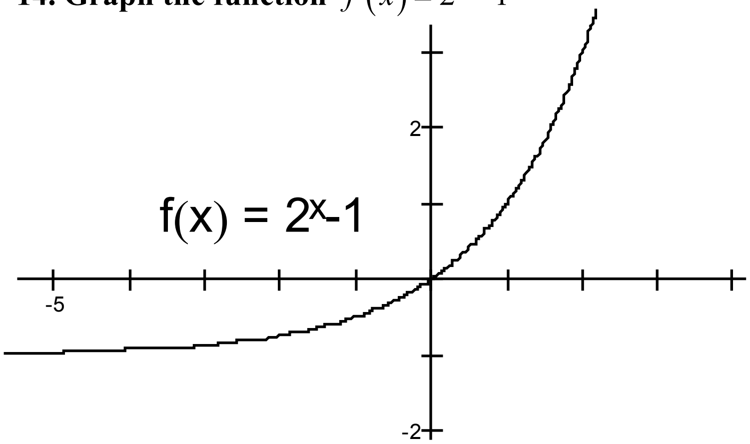
$\log_6 8 + \log_6 9 - \log_6 2 = \log_6 \frac{8 \cdot 9}{2} = \log_6 36 = \log_6 6^2 = 2\log_6 6 = 2$

13. Solve for *x* $\log_5 3x + 2 = 4$

$\log_5 3x + 2 = 4 \to 5^4 = 3x + 2$

$3x + 2 = 625 \to 3x = 623 \to x = \frac{623}{3}$

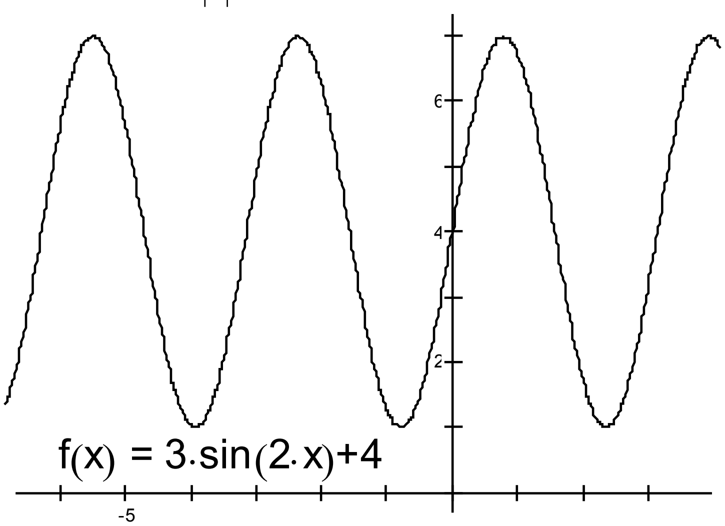
14. Graph the function $f(x) = 2^x - 1$



13. What are the domain, range, period and amplitude of $f(x) = 3\sin(2x) + 4$ **?**
Graph the function.

The domain is all real numbers, The range is $[4-3, 4+3] = [1, 7]$,

The period is $\frac{2\pi}{|2|} = \pi = 180^\circ$,The amplitude is 3



14. Put in the form $a+bi$**,** $\frac{3+2i}{5-i}$

$$\frac{3+2i}{5-i}\cdot\frac{5+i}{5+i}=\frac{15+10i+3i-2}{5^2+1^2}=\frac{13+13i}{26}=\frac{1}{2}+\frac{1}{2}i$$

15. Convert 80° **to radians**

$$80^\circ\cdot\frac{\pi}{180^\circ}=\frac{4}{9}\pi\approx 1.396 \text{ radians}$$

16. Find the exact value of

a) $\cos 120^\circ$

The reference angle for 120° which is in the 2nd quadrant is $180^\circ-120^\circ=60^\circ$

$\cos 60^\circ$ is a special angle whose value is $\frac{1}{2}$

The sign of cosine in the 2nd quadrant is negative, so $\cos 60^\circ=-\frac{1}{2}$

b) $\tan 390^\circ$

All trigonometric functions have period 360° so $\cos 120^\circ=-\frac{1}{2}$

$\tan 390^\circ=\tan 390^\circ-360^\circ=\tan 30^\circ$

$\tan 30^\circ$ is a special angle whose value is $\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}$ so $\tan 390^\circ=\frac{1}{\sqrt{3}}$

c) $\sec^- 45^\circ$

$$\sec^- 45^\circ=\frac{1}{\cos^- 45^\circ}=\frac{1}{\cos 45^\circ}$$

$\cos 45^\circ$ is a special angle whose value is $\frac{1}{\sqrt{2}}$ so

$$\sec^- 45^\circ=\frac{1}{1/\sqrt{2}}=\sqrt{2}$$

d) $\sin\frac{7\pi}{4}$

$\frac{7\pi}{4}$ is in the 4th quadrant, so the reference angle is $2\pi-\frac{7\pi}{4}=\frac{\pi}{4}$

$\sin\frac{\pi}{4}$ is a special angle whose value is $\frac{1}{\sqrt{2}}$. The sign of sine in the 4th quadrant is negative. So $\sin\frac{7\pi}{4}=-\frac{1}{\sqrt{2}}$

17. Solve the following triangles. Keep the solutions exact when possible.

a) $a = 10, b = 15, \angle c = 90^\circ$

Since this is a right triangle, the Pythagorean theorem gives the third side

$c = \sqrt{10^2 + 15^2} = \sqrt{325} = 5\sqrt{65} \approx 18.0$

Angle a can be found using an inverse trig function

$\angle a = \tan^{-1}\dfrac{10}{15} \approx 33.69^\circ$

Angle b is the complement of angle a

$\angle b \approx 90^\circ - 33.69^\circ = 56.31^\circ$

b) $a = 5, b = 7, \angle c = 45^\circ$

Using the law of cosines

$c = \sqrt{5^2 + 7^2 - 2 \cdot 5 \cdot 7 \cos 45^\circ} \approx 4.95$

Angle a can be found using the law of sines

$\dfrac{\sin \angle a}{5} = \dfrac{\sin 45^\circ}{4.95} \to \angle a = \sin^{-1}\left(\dfrac{5 \sin 45^\circ}{4.95}\right) \approx 45.58^\circ$

The final angle b can be found using the sum of angles in a triangle

$\angle b = 180^\circ - 45^\circ - 45.58^\circ \approx 89.42^\circ$

c) $a = 9, b = 11, c = 15$

Using the law of cosines

$\angle c = \cos^{-1}\left(\dfrac{15^2 - 9^2 - 11^2}{-2 \cdot 9 \cdot 11}\right) \approx 117.69^\circ$

Using the law of sines

$\dfrac{\sin \angle a}{9} = \dfrac{\sin 117.69}{15} \to \angle a = \sin^{-1}\left(\dfrac{9 \sin 117.69}{15}\right) \approx 32.09$

The final angle b can be found using the sum of angles in a triangle

$\angle b = 180^\circ - 117.69^\circ - 32.09^\circ \approx 30.22^\circ$

d) $a = 10, b = 14, \angle A = 30^\circ$

Using the law of sines

$\dfrac{\sin 30^\circ}{10} = \dfrac{\sin \angle B}{14} \to \angle B = \sin^{-1}\left(\dfrac{14 \sin 30^\circ}{10}\right) \approx 44.43^\circ$

The final angle c can be found using the sum of angles in a triangle

$\angle c = 180^\circ - 30^\circ - 44.43^\circ \approx 105.57^\circ$

The final side can be found using the law of sines

$$\frac{\sin 30^\circ}{10} = \frac{\sin \angle B}{14} \to \angle B = \sin^{-1}\left(\frac{14 \sin 30^\circ}{10}\right) \approx 44.43^\circ$$

18. Verify the identity $\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = 1$

$$\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x} = \frac{1/\cos x}{\cos x} - \frac{\sin x/\cos x}{\cos x/\sin x} = \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x} =$$

$$\frac{1-\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = 1$$

19. Find the exact value of $\cos\left(15^\circ\right)$

$$\cos\left(15^\circ\right) = \cos\frac{30^\circ}{2} = \pm\sqrt{\frac{1+\cos 30^\circ}{2}} = \pm\sqrt{\frac{1+\sqrt{3}/2}{2}} = \pm\sqrt{\frac{2+\sqrt{3}}{4}} =$$

$$\pm\frac{\sqrt{2+\sqrt{3}}}{2}$$

Since 15° is in the first quadrant $\cos\left(15^\circ\right) = \frac{\sqrt{2+\sqrt{3}}}{2}$

20. Solve this equation $6\sin^2 x - 5\sin x + 1 = 0$

$y = \sin x$

$6y^2 - 5y + 1 = 0$

$(3y-1)(2y-1) = 0$

$y = \frac{1}{3}, \frac{1}{2}$

$\sin x = \frac{1}{3} \to x = \sin^{-1}\left(\frac{1}{3}\right) \approx 19.47^\circ$

But also $x = 360^\circ - 19.47^\circ = 340.53^\circ$

$\sin x = \frac{1}{2} \to x = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

But also $x = 360^\circ - 30^\circ = 330^\circ$

So the solutions are:

$x = 19.47^\circ + 360^\circ n$

$x = 340.53^\circ + 360^\circ n$

$x = 30^\circ + 360^\circ n$

$x = 330^\circ + 360^\circ n$