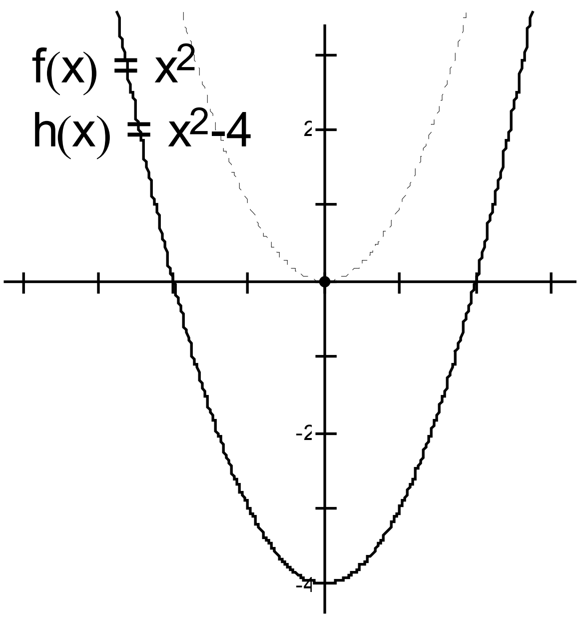
Answer Key 8
3.2: 5, 9-14, 16, 29, 30, 36, 39, 54
3.3: 3, 8, 9, 13, 23, 32, 35, 38 45, 46, 53, 57, 62, 67, 71, 74
3.4: 7, 13, 17, 26, 33, 40, 45, 53, 55, 57, 58, 81, 87, 105

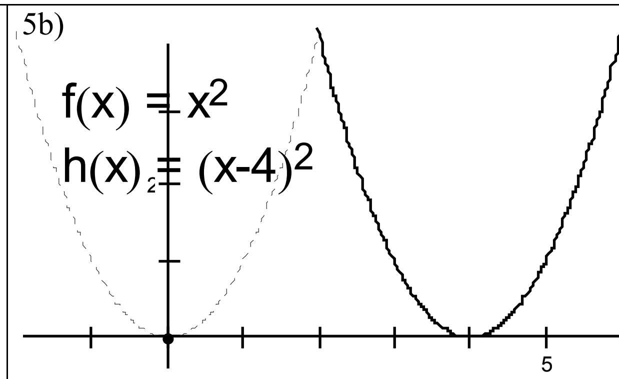
3.2

5a)



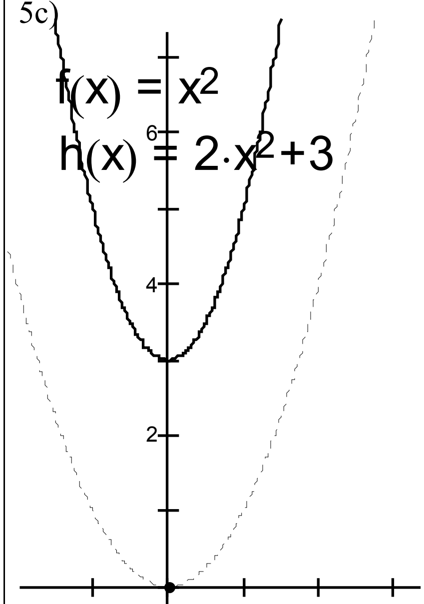
X-Intercepts 2,-2
Y-Intercept -4

5b)



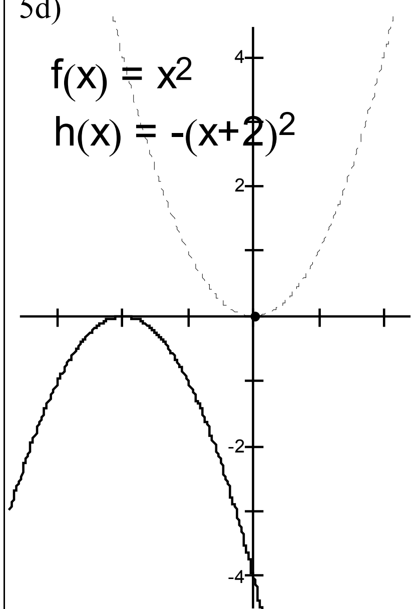
X-Intercept 4
Y-Intercept 16

5c)

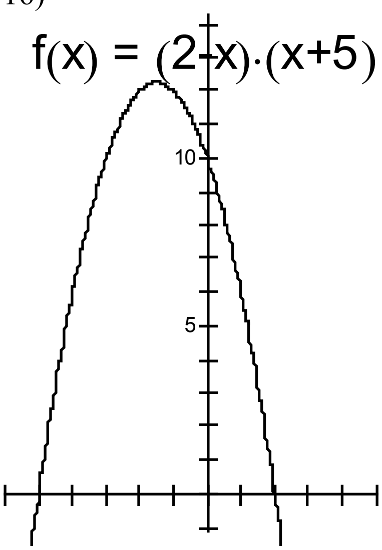
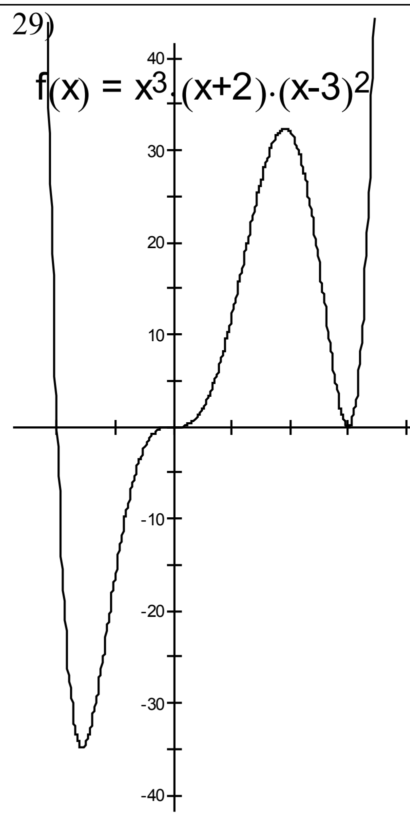


Y-Intercept 3

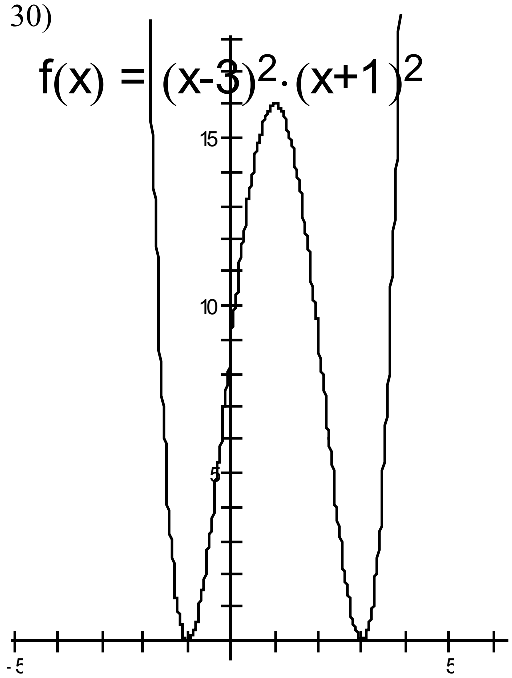
5d)



X-Intercept -2
Y-Intercept -4

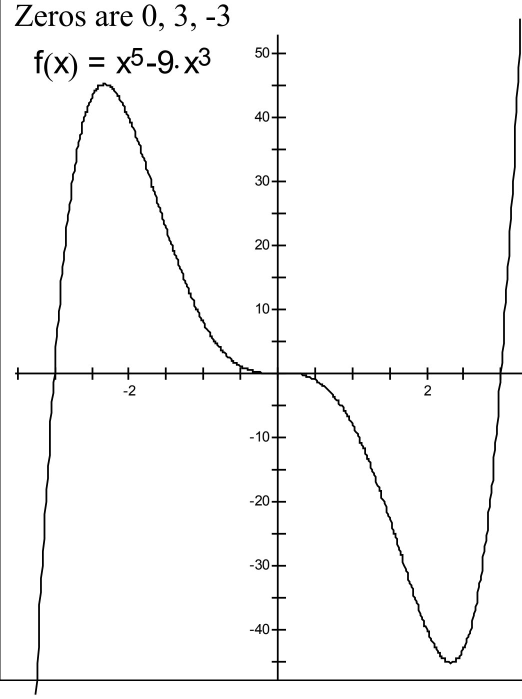
9) $P\left(x\right)=x\left(x^2-4\right)=x^3-4x$ a) $as\ x\rightarrow\infty\ P\left(x\right)\rightarrow\infty$ $as\ x\rightarrow-\infty\ P\left(x\right)\rightarrow-\infty$ b) III	10) $Q\left(x\right)=-x^2\left(x^2-4\right)=-x^4-4x^2$ a) $as\ x\rightarrow\infty\ P\left(x\right)\rightarrow-\infty$ $as\ x\rightarrow-\infty\ P\left(x\right)\rightarrow-\infty$ b) I
11) $R\left(x\right)=-x^5+5x^3-4x$ a) $as\ x\rightarrow\infty\ P\left(x\right)\rightarrow-\infty$ $as\ x\rightarrow-\infty\ P\left(x\right)\rightarrow\infty$ b) V	12) $S\left(x\right)=\frac{1}{2}x^6-2x^4$ a) $as\ x\rightarrow\infty\ P\left(x\right)\rightarrow\infty$ $as\ x\rightarrow-\infty\ P\left(x\right)\rightarrow\infty$ b) II
13))$T\left(x\right)=x^4-2x^3$ a) $as\ x\rightarrow\infty\ P\left(x\right)\rightarrow\infty$ $as\ x\rightarrow-\infty\ P\left(x\right)\rightarrow\infty$ b) VI	14))$U\left(x\right)=-x^3+2x^2$ a) $as\ x\rightarrow\infty\ P\left(x\right)\rightarrow-\infty$ $as\ x\rightarrow-\infty\ P\left(x\right)\rightarrow\infty$ b) IV
16)  	29) 

30)



36) $x^5 - 9x^3 = x^3\left(x^2 - 9\right) = x^3\left(x - 3\right)\left(x + 3\right)$

Zeros are 0, 3, -3



3.3

3)

$$\begin{array}{r|rrr} 2 & 2 & -5 & -7 \\ & & 4 & -2 \\ \hline & 2 & -1 & -9 \end{array}$$

$$\frac{2x^2 - 5x - 7}{x - 2} = 2x - 1 + \frac{-9}{x - 2}$$

8)

Using Long division

$$\frac{2x^5 + x^3 - 2x^2 + 3x - 5}{x^2 - 3x + 1} =$$

$$2x^3 + 6x^2 + 17x + 43 + \frac{115x - 48}{x^2 - 3x + 1}$$

9)

$$\begin{array}{r|rrrr} -1 & -1 & 0 & -2 & 6 \\ & & 1 & -1 & 3 \\ \hline & -1 & 1 & -3 & 9 \end{array}$$

$$-x^3 - 2x + 6 = \left(x + 1\right)\left(-x^2 + x - 3\right) + 9$$

13)

$$8x^4 + 4x^3 + 6x^2 =$$

$$\left(2x^2 + 1\right)\left(4x^2 + 2x + 2\right) + \left(-2x - 1\right)$$

23)

$$\frac{x^6 + x^4 + x^2 + 1}{x^2 + 1} = \frac{x^4\left(x^2 + 1\right) + x^2 + 1}{x^2 + 1} =$$

$$\frac{\left(x^4 + 1\right)\left(x^2 + 1\right)}{x^2 + 1} = x^4 + 1$$

32)

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & 1 & -1 & 2 \\ & & 2 & 2 & 6 & 10 \\ \hline & 2 & 1 & 3 & 5 & 12 \end{array}$$

$$\frac{x^4 - x^3 + x^2 - x + 2}{x - 2} =$$

$$x^3 + x^2 + 3x + 12$$

35) $\begin{array}{r\|rrrr} 1/2 & 2 & 3 & -2 & 1 \\ & & 1 & 2 & 0 \\ & 2 & 4 & 0 & 1 \end{array}$ quotient $2x^2+4$; remainder 1	38) $\dfrac{x^4-16}{x+2}=\dfrac{\left(x^2-4\right)\left(x^2+4\right)}{x+2}=$ $\dfrac{(x-2)(x+2)\left(x^2+4\right)}{x+2}=$ $(x-2)\left(x^2+4\right)=$ x^3-2x^2+4x-8
45) $\begin{array}{r\|rrrrr} -7 & 5 & 30 & -40 & 36 & 14 \\ & & -35 & 35 & 35 & -497 \\ & 5 & -5 & -5 & 71 & -483 \end{array}$ $P(-7)=-483$	46) $\begin{array}{r\|rrrrrr} -2 & 6 & 0 & 10 & 0 & 1 & 1 \\ & & -12 & 24 & -68 & 136 & -275 \\ & 6 & -12 & 34 & -68 & 137 & -273 \end{array}$ $P(-2)=-273$
53) $P(-2)=1^3-3\cdot 1^2+3\cdot 1-1=1-3+3-1=0$ *or* $P(x)=x^3-3x^2+3x-1=(x-1)^3$, so $P(1)=0$	57) $P(-2)=(-2)^3+2(-2)^2-9(-2)-18=$ $-8+8+18-18=0$ $\begin{array}{r\|rrrr} -2 & 1 & 2 & -9 & -18 \\ & & -2 & 0 & 18 \\ & 1 & 0 & -9 & 0 \end{array}$ $x^3-2x^2-9x-18=(x+2)\left(x^2-9\right)=$ $(x+2)(x+3)(x-3)$ other zeros are 3,-3

62)

$P(x) = 2x^4 - 13x^3 + 7x^2 + 37x + 15$

$P(-1) = 2 + 13 + 7 - 37 + 15 = 37 - 37 = 0$

$P(3) = 162 - 351 + 63 + 111 + 15 = 351 - 351 = 0$

$$\begin{array}{c|ccccc} -1 & 2 & -13 & 7 & 37 & 15 \\ & & -2 & 15 & -11 & -15 \\ & 2 & -15 & 22 & 15 & 0 \end{array}$$

$2x^4 - 13x^3 + 7x^2 + 37x + 15 = (x+1)(2x^3 - 15x^2 + 22x + 15)$

$$\begin{array}{c|cccc} 3 & 2 & -15 & 22 & 15 \\ & & 6 & -27 & -15 \\ & 2 & -9 & -5 & 0 \end{array}$$

$(x+1)(2x^3 - 15x^2 + 22x + 15) = (x+1)(x-3)(2x^2 - 9x - 5)$

Using the Quadratic formula

$x = \frac{9 \pm \sqrt{81+40}}{4} = \frac{9 \pm \sqrt{121+40}}{4} = \frac{9 \pm 11}{4} = 5, -\frac{1}{2}$

67) $P(x) = Ax(x+2)(x-1)(x-3) =$ $A(x^4 - 2x^3 - 2x^2 + 3x)$ $where\ A = -2$ $P(x) = -2(x^4 - 2x^3 - 2x^2 + 3x) =$ $-2x^4 + 4x^3 + 4x^2 - 12x$	71) $P(x) = A(x+1)(x-1)(x-2) =$ $A(x^3 - 2x^2 - x + 2)$ $P(0) = A(2) = 2$ $so\ A = 1$ $P(x) = x^3 - 2x^2 - x + 2$
74) $P(x) = A(x+2)(x+1)(x-1)^2 =$ $P(0) = A(2)(1)(-1)^2 = 2A = 2\ so\ A = 1$ $P(x) = (x+2)(x+1)(x-1)^2 =$ $x^4 + x^3 - 3x^2 - x + 2$	

3.4

7) $R(x)=2x^5+3x^3+4x^2-8$ $\frac{1\cdot 2\cdot 2\cdot 2}{1\cdot 2}\rightarrow \pm 1,\pm\frac{1}{2},\pm 2,\pm 4,\pm 8$	13) $P(x)=2x^4-9x^3+9x^2+x-3$ $\frac{1\cdot 3}{1\cdot 2}\rightarrow \pm 1,\pm 3,\pm\frac{1}{2},\pm\frac{3}{2}$ $-\frac{1}{2},1,3$
17) $P(x)=x^3+3x^2-4$ $\frac{1\cdot 2\cdot 2}{1}\rightarrow \pm 1,\pm 2,\pm 4$ $P(1)=1+3-4=0$ $\begin{array}{l\|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ & 1 & 4 & 4 & 0 \end{array}$ $x^3+3x^2-4=(x-1)(x^2+4x+4)=$ $(x-1)(x+2)^2$ Zeros are 1,-2	

26)

$P(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$

Possible integer rational roots are:

$\pm 1, \pm 2, \pm 4$

Trying them in order we find $P(1) = 0$ so 1 is a root

$$\begin{array}{c|ccccc} 1 & 1 & -2 & -3 & +8 & -4 \\ & & 1 & -1 & -4 & 4 \\ \hline & 1 & -1 & -4 & 4 & 0 \end{array}$$

$x^4 - 2x^3 - 3x^2 + 8 - 4 = (x-1)(x^3 - x^2 - 4x + 4)$

Possible integer rational roots now are:

$\pm 1, \pm 2, \pm 4$

Trying them in order we find

$P(1) = 0$ so 1 is a root again

$$\begin{array}{c|cccc} 1 & 1 & -1 & -4 & 4 \\ & & 1 & 0 & -4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

$(x-1)(x^3 - x^2 - 4x + 4) = (x-1)(x-1)(x^2 - 4)$

Factoring we get

$(x-1)(x-1)(x-2)(x+2)$

so the roots are 1, 2, -2 with 1 having multiplicity 2.

33)

$P(x) = 4x^3 + 4x^2 - x - 1$

Possible integer rational roots are:

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}$

Trying them in order we find

$P(-1) = 0$ so -1 is a root.

$$\begin{array}{c|cccc} -1 & 4 & 4 & -1 & -1 \\ & & -4 & 0 & 1 \\ \hline & 4 & 0 & -1 & 0 \end{array}$$

$4x^3 + 4x^2 - x - 1 = (x+1)(4x^2 - 1) = (x+1)(2x+1)(2x-1)$

Zeros are $-1, \pm \frac{1}{2}$

40)

$P(x)=6x^4-7x^3-12x^2+3x+2$

Possible rational roots are: $\pm\frac{1}{1},\pm\frac{2}{1},\pm\frac{1}{2},\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{1}{6}$

Trying them in order we find $P(-1)=0$ so -1 is a root

$$\begin{array}{r|rrrrr} -1 & 6 & -7 & -12 & 3 & 2 \\ & & -6 & 13 & -1 & -2 \\ & 6 & -13 & 1 & 2 & 0 \end{array}$$

$6x^4-7x^3-12x^2+3x+2=(x+1)(6x^3-13x^2+x+2)$

Possible rational roots are now: $\pm\frac{2}{1},\pm\frac{1}{2},\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{1}{6}$

Trying them in order we find $P(2)=0$ so 2 is a root

$$\begin{array}{r|rrrr} 2 & 6 & -13 & 1 & 2 \\ & & 12 & -2 & -2 \\ & 6 & -1 & -1 & 0 \end{array}$$

$(x+1)(6x^3-13x^2+x+2)=(x+1)(x-2)(6x^2-x-1)$

Using the quadratic formula

$$x=\frac{1\pm\sqrt{1+24}}{12}=\frac{1\pm 5}{12}=\frac{1}{2},-\frac{1}{3}$$

So the roots are $-1,2,\frac{1}{2},-\frac{1}{3}$

45)

$P(x)=3x^3+5x^2-2x-4$

Possible rational roots are: $\pm\frac{1}{1},\pm\frac{2}{1},\pm\frac{4}{1},\pm\frac{1}{3},\pm\frac{2}{3},\pm\frac{4}{3}$

Trying them in order we find $P(-1)=0$ so -1 is a root

$$\begin{array}{r|rrrr} -1 & 3 & 5 & -2 & -4 \\ & & -3 & -2 & 4 \\ & 3 & 2 & -4 & 0 \end{array}$$

$3x^3+5x^2-2x-4=(x+1)(3x^2+2x-4)$

Using the quadratic formula

$$x=\frac{-2\pm\sqrt{4+48}}{6}=\frac{-2\pm\sqrt{52}}{6}=\frac{-2\pm 2\sqrt{14}}{6}=\frac{-1\pm\sqrt{14}}{3}$$

So the roots are $-1,\frac{-1\pm\sqrt{14}}{3}$

53)

$P(x) = 2x^4 + 15x^3 + 17x^2 + 3x - 1$

Possible rational roots are: $\pm\frac{1}{1}, \pm\frac{1}{2}$

Trying them in order we find $P(-1) = 0$ so -1 is a root

-1	2	15	17	3	-1
		-2	-13	-4	1
	2	13	4	-1	0

$2x^4 + 15x^3 + 17x^2 + 3x - 1 = (x+1)(2x^3 + 13x^2 + 4x - 1)$

Possible rational roots are now: $-1, \pm\frac{1}{2}$

Trying them in order we find $P\left(\frac{-1}{2}\right) = 0$ so -1/2 is a root

$-1/2$	2	13	4	-1
		-1	-6	1
	2	12	-2	0

$2x^4 + 15x^3 + 17x^2 + 3x - 1 = (x+1)(x+1/2)(2x^2 + 12x - 2) =$

$(x+1)(2x+1)(x^2 + 6x - 1)$

Using the quadratic formula

$$x = \frac{-6 \pm \sqrt{36+4}}{2} = \frac{-6 \pm \sqrt{40}}{2} = \frac{-6 \pm 2\sqrt{10}}{2} = -3 \pm \sqrt{10}$$

So the roots are $-1, -\frac{1}{2}, -3 \pm \sqrt{10}$

55)

$P(x) = x^3 - 3x^2 - 4x + 12$

Possible rational roots are: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Trying them in order we find $P(2) = 0$ so 2 is a root

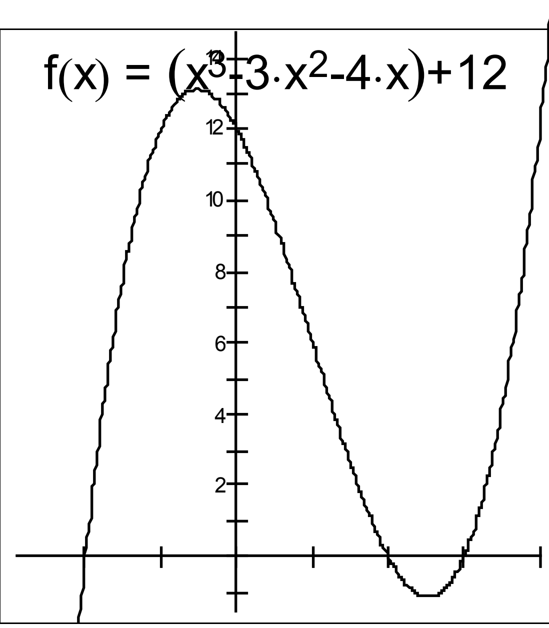
2	1	-3	-4	12
		2	-2	-12
	1	-1	-6	0

$x^3 - 3x^2 - 4x + 12 = (x-2)(x^2 - x - 6)$

Factoring

$(x-2)(x^2 - x - 6) = (x-2)(x+2)(x-3)$

So the roots are 2, -2, and 3



57)

$P(x) = 2x^3 - 7x^2 + 4x + 4$

Possible rational roots are: $\pm 1, \pm 2, \pm 4$

Trying them in order we find $P(2) = 0$

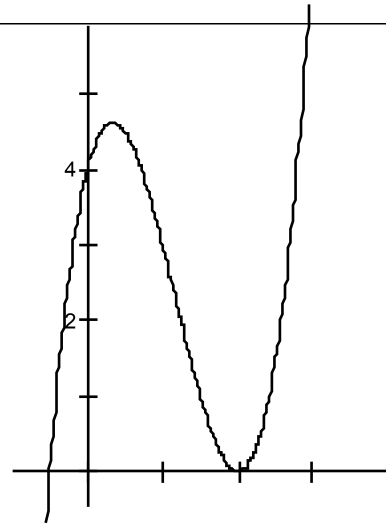
so 2 is a root

$$\begin{array}{c|cccc} 2 & 2 & -7 & 4 & 4 \\ & & 4 & -6 & -4 \\ & 2 & -3 & -2 & 0 \end{array}$$

$2x^3 - 7x^2 + 4x + 4 = (x-2)(2x^2 - 3x - 2) =$

$(x-2)(x-2)(x+1)$

So the roots are 2 and -1



58)

$P(x) = 3x^3 + 17x^2 + 21x - 9$

Possible rational roots are: $\pm 1, \pm \frac{1}{3} \pm 3, \pm 9$

Trying them in order we find $P(1/3) = 0$

so 1/3 is a root

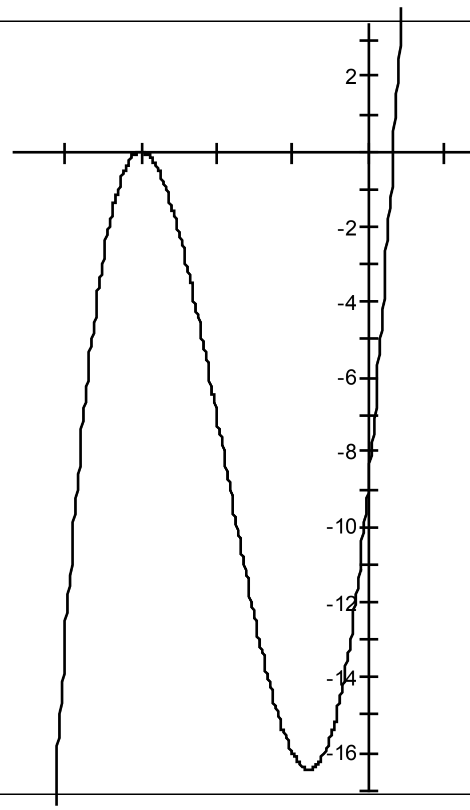
$$\begin{array}{c|cccc} 1/3 & 3 & 17 & 21 & -9 \\ & & 1 & 6 & 9 \\ & 3 & 18 & 27 & 0 \end{array}$$

$3x^3 + 17x^2 + 21x - 9 =$

$(x - 1/3)(3x^2 + 18x + 27) =$

$(3x-1)(x^2 + 6x + 9) = (3x-1)(x+3)^2$

So the roots are 1/3, -3



81)

$P(x) = 2x^4 + 3x^3 - 4x^2 - 3x + 2$

Possible rational roots are: $\pm 1, \pm 2, \pm \frac{1}{2}$

Trying them in order we find $P(1) = 0$

so 1 is a root

$$\begin{array}{c|ccccc} 1 & 2 & 3 & -4 & -3 & 2 \\ & & 2 & 5 & 1 & -2 \\ \hline & 2 & 5 & 1 & -2 & 0 \end{array}$$

$2x^4 + 3x^3 - 4x^2 - 3x + 2 =$

$(x-1)(2x^3 + 5x^2 + x - 2)$

All roots are still Possible

Trying them in order we find $P(-1) = 0$

so -1 is a root

$$\begin{array}{c|cccc} -1 & 2 & 5 & 1 & -2 \\ & & -2 & -3 & 2 \\ \hline & 2 & 3 & -2 & 0 \end{array}$$

$(x-1)(2x^3 + 5x^2 + x - 2) =$

$(x-1)(x+1)(2x^2 + 3x - 2)$

Using the quadratic formula

$$x = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm \sqrt{25}}{4} =$$

$$\frac{-3 \pm 5}{4} = -2, \frac{1}{2}$$

So the roots are $1, -2, -2, \frac{1}{2}$

87)

$P(x) = x^3 - x - 2$

Possible rational roots are:

$\pm 1, \pm 2$

$P(1) = 1 - 1 - 2 = -2$

$P(-1) = -1 + 1 - 2 = -2$

$P(2) = 8 - 2 - 2 = 4$

$P(-2) = -8 + 2 - 2 = -8$

These are the only possible rational roots and none of them are roots, so the polynomial does not have any rational roots.

105)

$Girth = 4b$

$l + Girth = 108$

$l = 108 - Girth = 108 - 4b$

$Volume = lb^2 = 2200$

$(108 - 4b)b^2 = 2200$

$4b^3 - 108b^2 + 2200 = 0$

$b^3 - 27b^2 + 550 = 0$

Looking at the graph of this function, you can see it has a root at 5.

Dividing you can find that

$b^3 - 27b^2 + 550 = (b-5)(b^2 - 22b - 110)$

Using the Quadratic formula you can find

$b = 11 \pm \sqrt{231} \approx 26.2, -4.2$

Only the positive roots make sense,

5 and 26.2 so the length can be

88 or 3.2