Answer Key 15

7.2: 5-8, 32,35, 43
7.3: 5-8, 17, 21, 25, 55, 57, 60, 61, 63, 66, 73, 74
7.2

5) $\cos\left(105^\circ\right)=\cos\left(45^\circ+60^\circ\right)$ $=\cos\left(45^\circ\right)\cos\left(60^\circ\right)-\sin\left(45^\circ\right)\sin\left(60^\circ\right)=$ $\frac{1}{\sqrt{2}}\frac{1}{2}+\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2\sqrt{2}}=\frac{\sqrt{2}+\sqrt{6}}{4}$	6) $\cos\left(195^\circ\right)=\cos\left(135^\circ+60^\circ\right)$ $=\cos\left(135^\circ\right)\cos\left(60^\circ\right)-\sin\left(135^\circ\right)\sin\left(60^\circ\right)=$ $\frac{-1}{\sqrt{2}}\frac{1}{2}+\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}=\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{6}-\sqrt{2}}{4}$
7) $\tan\left(15^\circ\right)=\tan\left(45^\circ-30^\circ\right)$ $=\frac{\tan\left(45^\circ\right)-\tan\left(30^\circ\right)}{1+\tan\left(45^\circ\right)\tan\left(30^\circ\right)}=$ $\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}\cdot\frac{1-\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}=\frac{1+\frac{1}{3}-\frac{2\sqrt{3}}{3}}{\frac{2}{3}}=$ $\frac{4-2\sqrt{3}}{2}=2-\sqrt{3}$	8) $\tan\left(165^\circ\right)=\tan\left(120^\circ+45^\circ\right)$ $=\frac{\tan\left(120^\circ\right)+\tan\left(45^\circ\right)}{1-\tan\left(120^\circ\right)\tan\left(45^\circ\right)}=$ $\frac{-\sqrt{3}+1}{1-{}^{-}\sqrt{3}}=\frac{1-\sqrt{3}}{1+\sqrt{3}}\cdot\frac{1-\sqrt{3}}{1-\sqrt{3}}=$ $\frac{1+3-2\sqrt{3}}{-2}=\frac{2\sqrt{3}-4}{2}=\sqrt{3}-2$
32) $\cos\left(x+\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{6}\right)=$ $\cos\left(x\right)\cos\left(\frac{\pi}{3}\right)-\sin\left(x\right)\sin\left(\frac{\pi}{3}\right)+$ $\sin\left(x\right)\cos\left(\frac{\pi}{6}\right)-\cos\left(x\right)\sin\left(\frac{\pi}{6}\right)=$ $\cos\left(x\right)\frac{1}{2}-\sin\left(x\right)\frac{\sqrt{3}}{2}+$ $\sin\left(x\right)\frac{\sqrt{3}}{2}-\cos\left(x\right)\frac{1}{2}=0$	35) $\sin\left(x+y\right)-\sin\left(x-y\right)=$ $\sin x\cos y+\cos x\sin y-$ $\left[\sin x\cos y-\cos x\sin y\right]=$ $\cos x\sin y-\left[-\cos x\sin y\right]=$ $2\cos x\sin y$
43) $\cos\left(x+y\right)\cos\left(x-y\right)=$ $\left(\cos x\cos y-\sin x\sin y\right)\cdot$ $\left(\cos x\cos y+\sin x\sin y\right)=$ $\left(\cos x\cos y\right)^2-\left(\sin x\sin y\right)^2=$ $\cos^2 x\cos^2 x-\sin^2 x\sin^2 y=$	$\cos^2 x\cos^2 x-\left(1-\cos^2 x\right)\sin^2 y=$ $\cos^2 x\cos^2 x+\cos^2 x\sin^2 y-\sin^2 y=$ $\cos^2 x\left(\cos^2 x+\sin^2 y\right)-\sin^2 y=$ $\cos^2 x-\sin^2 y$

7.3

5) $\cos x = 4/5$ $\sin x = \pm\sqrt{1-(4/5)^2} = \pm 3/5$ Since $1/\sin x < 0$ $\sin x = -3/5$ $\sin 2x = 2\sin x\cos x = -24/25$ $\cos 2x = 2\cos^2 x - 1 = 32/25 - 1 = 7/25$ $\tan 2x = \sin 2x/\cos 2x = -24/7$	6) $\csc x = 4 \rightarrow \sin x = 1/4$ $\cos x = \pm\sqrt{1-(1/4)^2} = \pm\sqrt{15}/4$ Since $\tan x < 0$ $\cos x = -\sqrt{15}/4$ $\sin 2x = 2\sin x\cos x = -\sqrt{15}/8$ $\cos 2x = 1 - 2\sin^2 x = 1 - 1/8 = 7/8$ $\tan 2x = \sin 2x/\cos 2x = -\sqrt{15}/7$
7) $\sin x = -3/5$ $\cos x = \pm\sqrt{1-(-3/5)^2} = \pm 4/5$ Since x is in quadrant III $\cos x = -4/5$ $\sin 2x = 2\sin x\cos x = 24/25$ $\cos 2x = 2\cos^2 x - 1 = 32/25 - 1 = 7/25$ $\tan 2x = \sin 2x/\cos 2x = 24/7$	8) $\sec x = 2 \rightarrow \cos x = 1/2$ $\sin x = \pm\sqrt{1-(1/2)^2} = \pm\sqrt{3}/2$ Since x is in quadrant IV $\sin x = -\sqrt{3}/2$ $\sin 2x = 2\sin x\cos x = -\sqrt{3}/2$ $\cos 2x = 2\cos^2 x - 1 = (1/2) - 1 = -1/2$ $\tan 2x = \sin 2x/\cos 2x = \sqrt{3}$
17) Note 15° is in quadrant I $\sin(15^\circ) = \sin\left(\frac{30^\circ}{2}\right) = \sqrt{\frac{1-\cos 30^\circ}{2}} =$ $\sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}}$	21) Note 165° is in quadrant II $\cos(330^\circ) = \cos(30^\circ) = \sqrt{3}/2$ $\cos(165^\circ) = \cos\left(\frac{330^\circ}{2}\right) =$ $-\sqrt{\frac{1+\cos 30^\circ}{2}} = -\sqrt{\frac{1+\sqrt{3}/2}{2}} = -\sqrt{\frac{2+\sqrt{3}}{4}}$
25) Note $\pi/12$ is in quadrant I $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi/6}{2}\right) = \sqrt{\frac{1+\cos\pi/6}{2}} =$ $\sqrt{\frac{1+\sqrt{3}/2}{2}} = \sqrt{\frac{2+\sqrt{3}/2}{4}}$	55) $\sin 2x\cos 3x = \frac{1}{2}\left[\sin 5x + \sin(-x)\right] =$ $\frac{1}{2}\left[\sin 5x - \sin x\right]$
57) $\cos x\sin 4x = \frac{1}{2}\left[\sin 5x - \sin(-3x)\right] =$ $\frac{1}{2}\left[\sin 5x + \sin 3x\right]$	60) $11\sin(x/2)\cos(x/4) = \frac{1}{2}\left[\sin(3x/4) + \sin(x/2)\right]$

61) $\sin 5x + \sin 3x = 2\sin 4x \cos x$	63) $\cos 4x - \cos 6x = -2\sin 5x \sin(-2x) =$ $2\sin 5x \sin 2x$
66) $\sin 3x + \sin 4x = 2\sin\left(\frac{7x}{2}\right)\cos\left(\frac{-x}{2}\right) =$ $2\sin\left(\frac{7x}{2}\right)\cos\left(\frac{x}{2}\right)$	73) Prove $\cos^2(5x) - \sin^2(5x) = \cos(10x)$ Substitute *5x=y* $\cos^2(5x) - \sin^2(5x) = \cos^2(y) - \sin^2(y)$ By the cosine double angle formula $\cos^2(y) - \sin^2(y) = \cos 2y$ Substituting *y=5x* $\cos 2y = \cos 10x$ So by transitivity $\cos^2(5x) - \sin^2(5x) = \cos(10x)$
74) Prove $\sin 8x = 2\sin 4x \cos 4x$ Substitute 4*x=y* $\sin 8x = \sin 2y$ By the sin double angle formula $\sin 2y = 2\sin y \cos y$ Substituting *y*=4*x* $2\sin y \cos y = 2\sin 4x \cos 4x$ So by transitivity $\sin 8x = 2\sin 4x \cos 4x$	