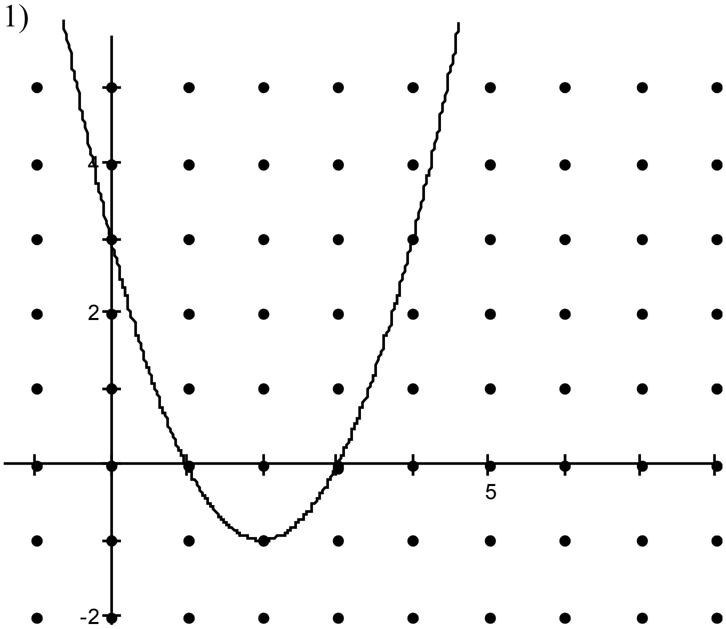
Answer Key Mid-Term 2

1)



2)
$$f(-x) = (-x)^4 + (-x)^2 = (-1)x^4 + (-1)x^2 = x^4 + x^2 = f(x)$$
so function is even

3)
$$f(-x) = (-x)^3 - 1 = (-1)x^3 - 1 = -x^3 - 1 \neq f(x)$$
function is neither

4) $(f+g)(3) = f(3) + g(3) = \sqrt{3+1} + 3 + 3 = \sqrt{4} + 6 = 2 + 6 = 8$

5) $(f \cdot g)(0) = f(0)g(0) = \sqrt{0+1} \cdot (0+3) = 1 \cdot 3 = 3$

6) $(f \circ g)(5) = f(g(5)) = \sqrt{(5+3)+1} = \sqrt{9} = 3$

7) Note this is a "disguised quadratic"
$$x^4 - 3x^2 + 2 = (x^2)^2 - 3(x^2) + 2 = (x^2 - 2)(x^2 - 1) =$$
$$(x - \sqrt{2})(x + \sqrt{2})(x - 1)(x + 1)$$
So the roots are $\pm 1, \pm\sqrt{2}$

8) Using Synthetic division

2	1	-1	0	3	2	-32
		2	2	4	14	32
	1	1	2	7	16	

So $\dfrac{x^5 - x^4 + 3x^2 - 2x - 32}{x-2} = x^4 + x^3 + 2x^2 + 7x + 16$

9)

Using the remainder theorem we know that

if $P(x) = (x-a)Q(x) + R$ that $R = P(a)$

so if $x^{40} + 5x^2 + 3 = (x-1)Q(x) + R$

then $R = 1^{40} + 5 \cdot 1^2 + 3 = 1 + 5 + 3 = 8$

10) Using the rational root theorem we know that the rational roots of $a_n x^n + \cdots a_0$ must be a factor of $\dfrac{a_0}{a_n}$.

So the rational roots of $2x^{23} - 3$ must be factors of $\dfrac{3}{2}$.

These are $\pm\dfrac{1}{1}, \pm\dfrac{3}{1}, \pm\dfrac{1}{2}, \pm\dfrac{3}{2}$.

11)

First factor out the common factor x^2

$x^5 - x^4 + 4x^3 - 4x^2 = x^2\left(x^3 - x^2 + 4x - 4\right)$

This gives us the root x=0 but with multiplicity 2.

This can be further factored either by using the rational root theorem and checking $\pm 1, \pm 2, \pm 4$. Note that 1 will be immediately found to be a root.

Using synthetic or long division $x^2\left(x^3 - x^2 + 4x - 4\right) = x^2(x-1)\left(x^2+4\right)$

Finally $x^2 + 4$ can be factored using the quadratic formula, noting that b=0 or by using the pattern $A^2 + B^2 = (A + Bi)(A - Bi)$

$x^2(x-1)\left(x^2+4\right) = x^2(x-1)(x+2i)(x-2i)$

So the roots are 0 (with multiplicity 2), 1, 2i and -2i

12)

Given the rational function $\frac{P(x)}{Q(x)}$ find the zeros by finding the zeros of *P*(*x*).

$x^2-2x+1=(x-1)^2$ so there is a zero at *x*=1

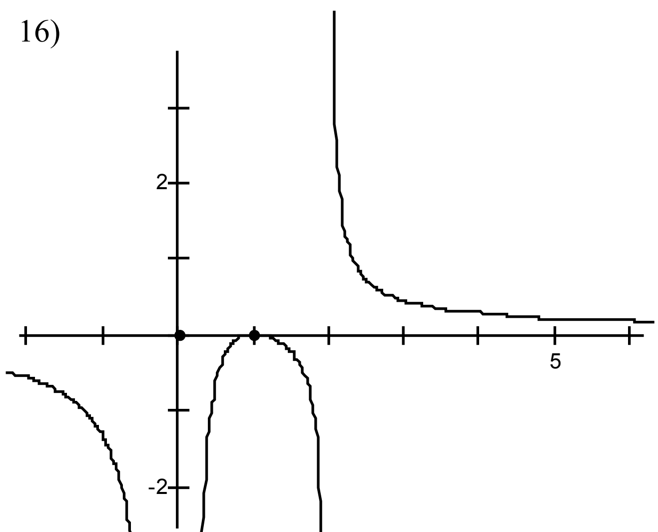
13)

Given the rational function $\frac{P(x)}{Q(x)}$ find the vertical asymptotes by finding the zeros of *Q*(*x*), and then verifying that there is no common factor with *P*(*x*).
$x^3-2x^2=x^2(x-2)$ so there will be vertical asymptotes at *x*=0, and *x*=2.

14) Horizontal asymptotes depend on the end behavior of the rational function.
Note that as *x* gets very large, $\frac{x^2-2x+1}{x^3-2x^2}$ gets close to $\frac{x^2}{x^3}=\frac{1}{x}$.
But $\frac{1}{x}$ will get closer and closer to 0, so there is a horizontal asymptote at *y*=0.

15) The *y* intercept is found at $f(0)=\frac{0^2-2\cdot 0+1}{0^3-2\cdot 0}=\frac{1}{0}$
But division by zero is undefined so the *y* intercept is undefined, or there isn't one.

16)



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Note the zero at (1,0), the vertical asymptotes at *x*=0 and *x*=2, and the horizontal asymptote at *y*=0.

17) A polynomial function whose roots are *a,b,c* and *d* has the form
$A(x-a)(x-b)(x-c)(x-d)$.
Using the conjugate root theorem we know that since 1-*i* is a root, 1+*i* must also be a root.
So the function can be $f(x)=A(x-1)(x-2)(x-(1-i))(x-(1+i))$ have any $A\neq 0$

18)

$$P_f(t) = P_0\left(1+\frac{i}{n}\right)^{nt}$$ where

$P_f(t)$ is the future principle

t is years

P_0 is the current principle

i is the interest rate

and *n* is the how often the interest is compounded yearly

$$P_f(t) = 800\left(1+\frac{.1}{4}\right)^{4\cdot 3} = 1075.91$$

19) $\log_6 1 = x \to 6^x = 1 \to x = 0$

20) $\log_7 49 = x \to 7^x = 49 \to x = 2$

21)

Using the law of logs

$$\log_4 20 + \log_4 16 - \log_4 5 = \log_4\left(\frac{20\cdot 16}{5}\right) = \log_4 64$$

$$\log_4 64 = x \to 4^x = 49 \to x = 3$$

22)

$$\log_3 2x = 1 \to 3^1 = 2x \to x = \frac{3}{2}$$

23)

$$5^{3x} = 25^{2x-1} \to 5^{3x} = \left(5^2\right)^{2x-1} \to 5^{3x} = 5^{4x-2}$$

Since the exponential function is one-to-one

$3x = 4x - 2 \to x = 2$

Alternative solution, take $\log_5$ on both sides of the equation

$$\log_5 5^{3x} = \log_5 25^{2x-1}$$
$$3x\left(\log_5 5\right) = \left(2x-1\right)\log_5 25$$
$$3x = \left(2x-1\right)\cdot 2$$
$$3x = 4x - 2$$
$$x = 2$$

24)

$f(x) = \sqrt{x+4}$

$y = \sqrt{x+4}$

Switch the x and y and solve for y

$x = \sqrt{y+4}$

$x^2 = y+4$

$y = x^2 - 4$

25)

Since the square root is undefined over the reals for $x<0$
the domain of $f(x)$ is $x+4 \geq 0$ *or* $x \geq -4$.
The range of the square root is $y \geq 0$

The domain and range of an inverse function will be the opposite of the function so the domain of $f^{-1}(x)$ is $x \geq 0$ and the range is $y \geq -4$.