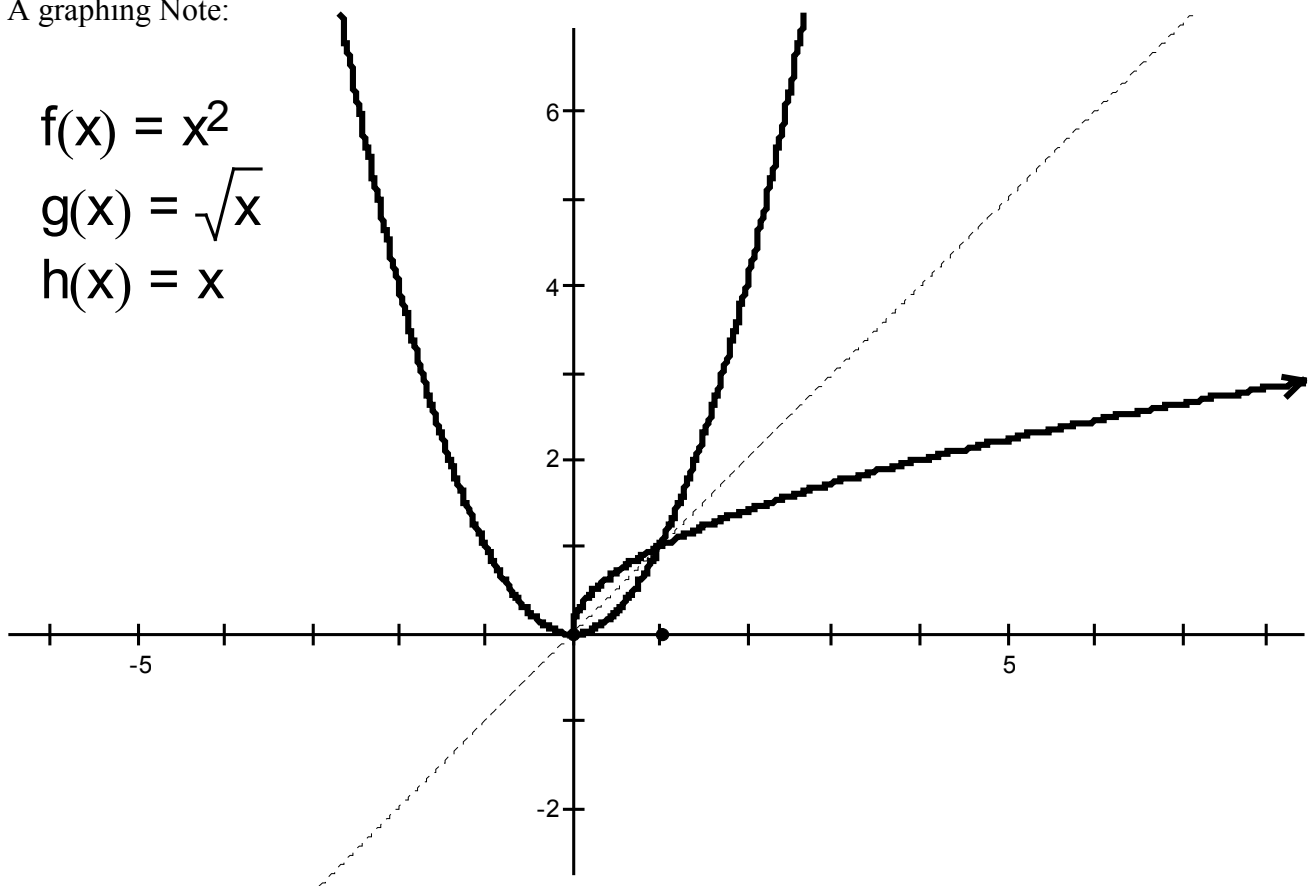**Inverse Trig Functions**
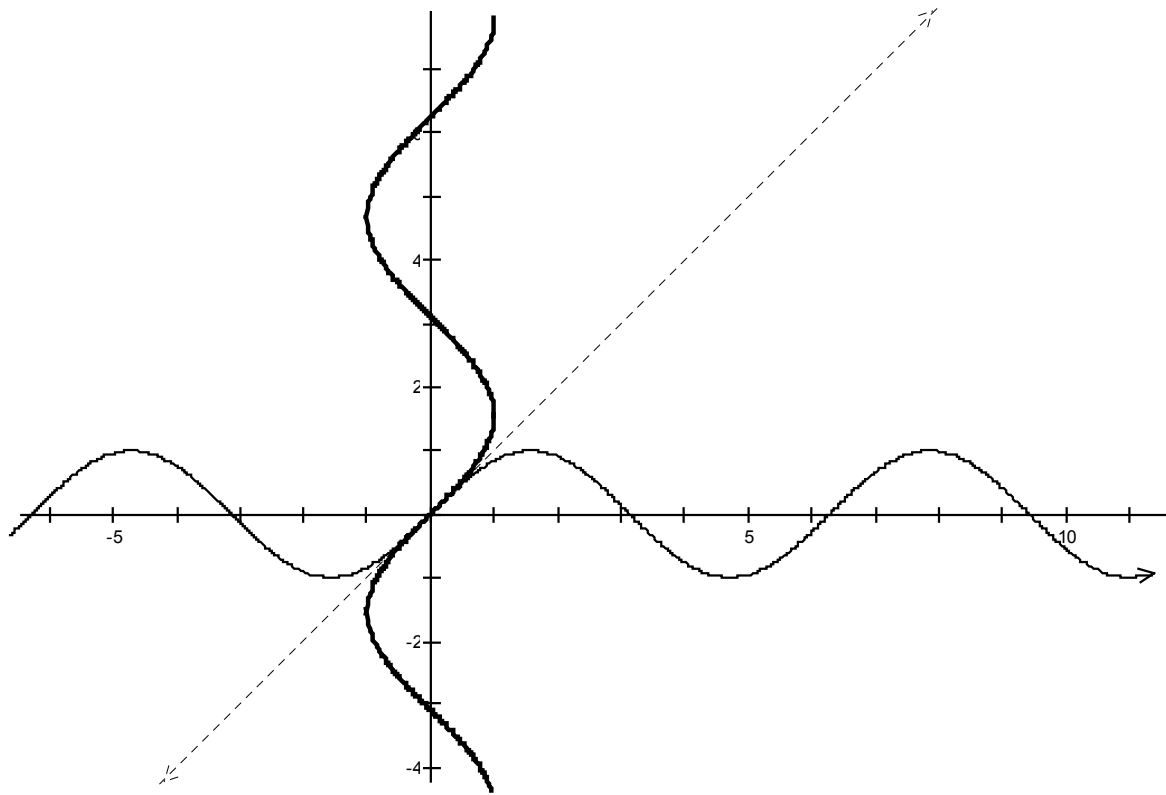
A graphing Note:

$$f(x) = x^2$$
$$g(x) = \sqrt{x}$$
$$h(x) = x$$

∟

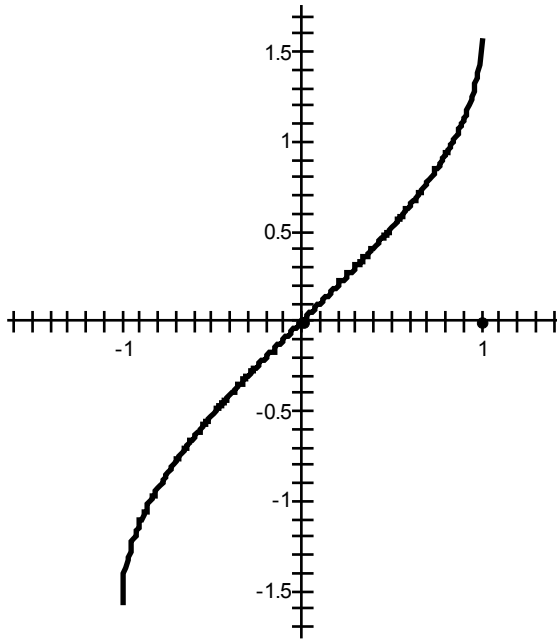You can graph the inverse function by rotating along the line y=x.

How did we need to truncate the domain to make the inverse a function?

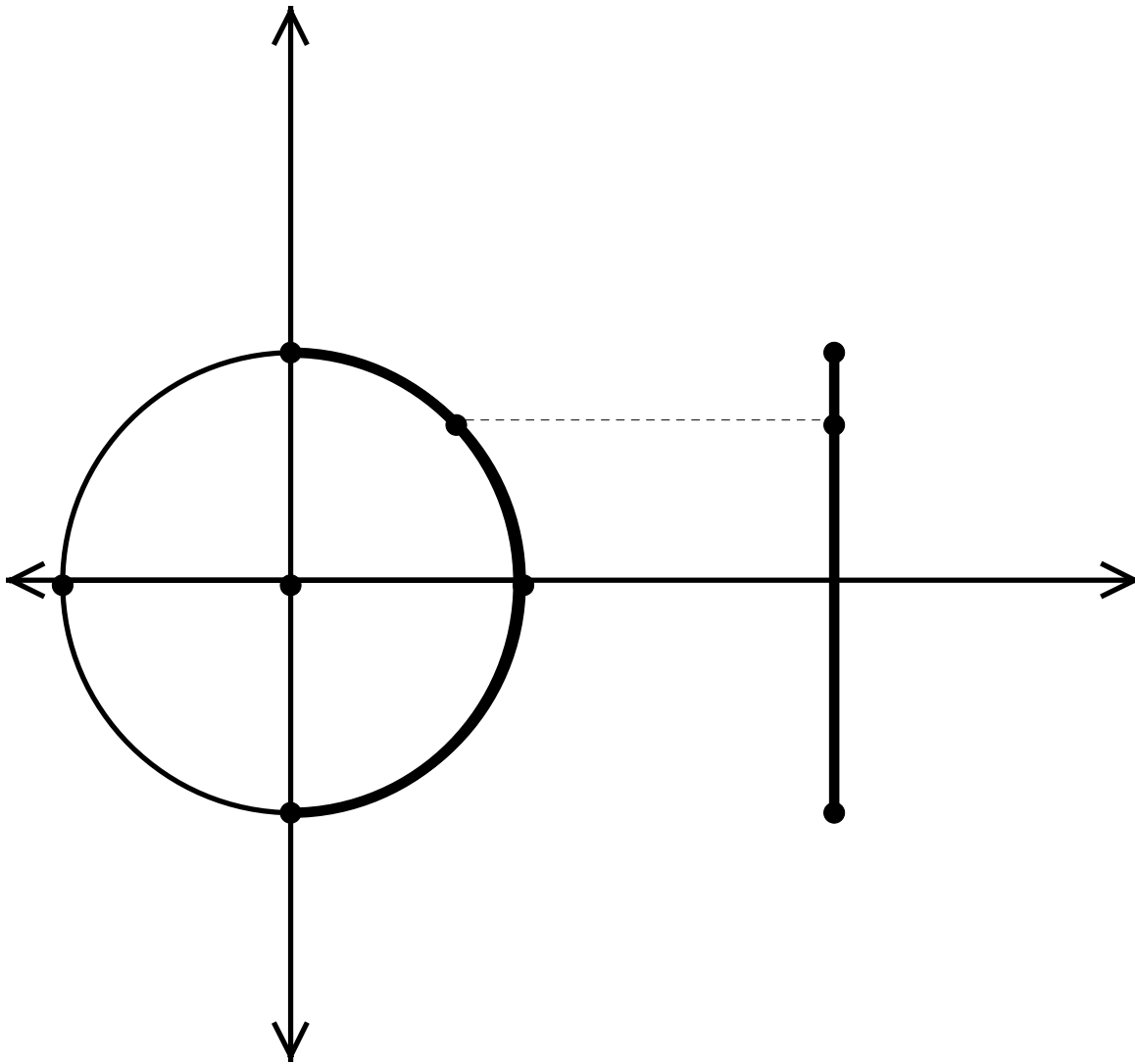If you do this with the sine function, the result is not a function anymore.

So we must truncate the Domain of Sine so that it's inverse exists.

We truncate the Domain to be $[-\pi, \pi]$

This is arbitrary, however it is the standard way of doing this.
You will note that this is what your calculator does.
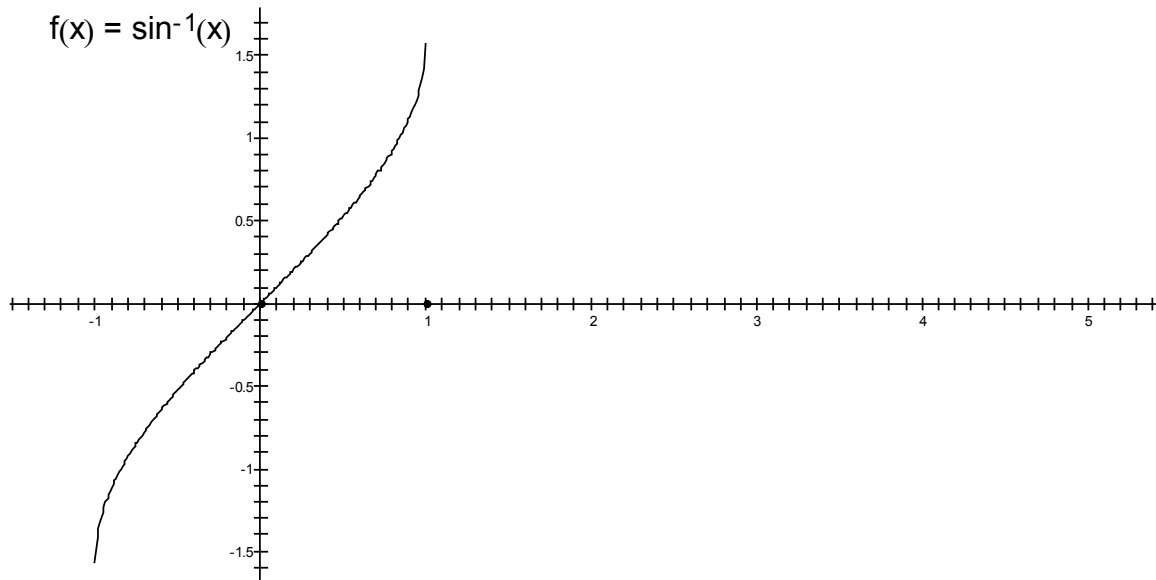
Back to The Sine Function and it's inverse.

We limit the Sine function to $[-\pi, \pi]$.

The inverse of the Sine function is written either

$\arcsin(x)$

or

$\sin^{-1}(x)$

$f(x) = \sin^{-1}(x)$

The domain of this function [-1, 1] is the range of it's inverse.

The range of this function $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$ is the domain of it's inverse.

You can use your calculator to find specific values of an inverse function:

Note that the resulting value will depend on the MODE of the calculator Degrees or Radians.

Some notes on Notation

While we are interested in an inverse to the Sine function, also of interest in an inverse relationship.

A relationship like a function is a mapping however it does not need to pass the vertical line test.   That is a relation can be one to many.

Thinking about the inverse Sine relation it would map

$0 \to 0$
$0 \to \pi$
$0 \to 2\pi$
$0 \to \pi n \; n \in \mathbb{Z}$

One way to make the distinction is with notation.

We use arcsin(x) to mean the inverse Sine function

We can use Arcsin(x) to mean the inverse Sine relation.

This also goes for

Arccos(x)
Arctan(x)
Arccot(x)
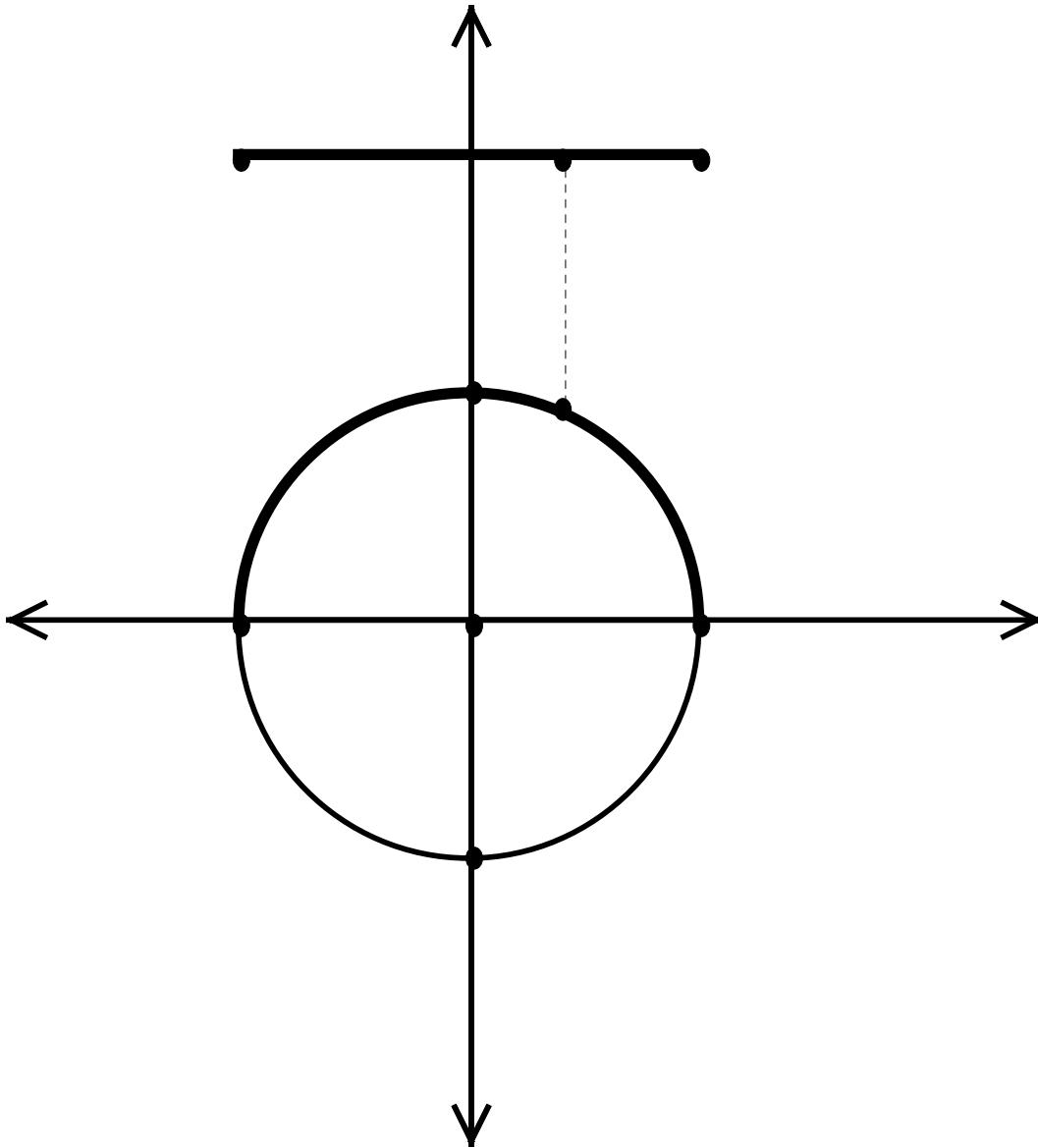Arcsec(x)
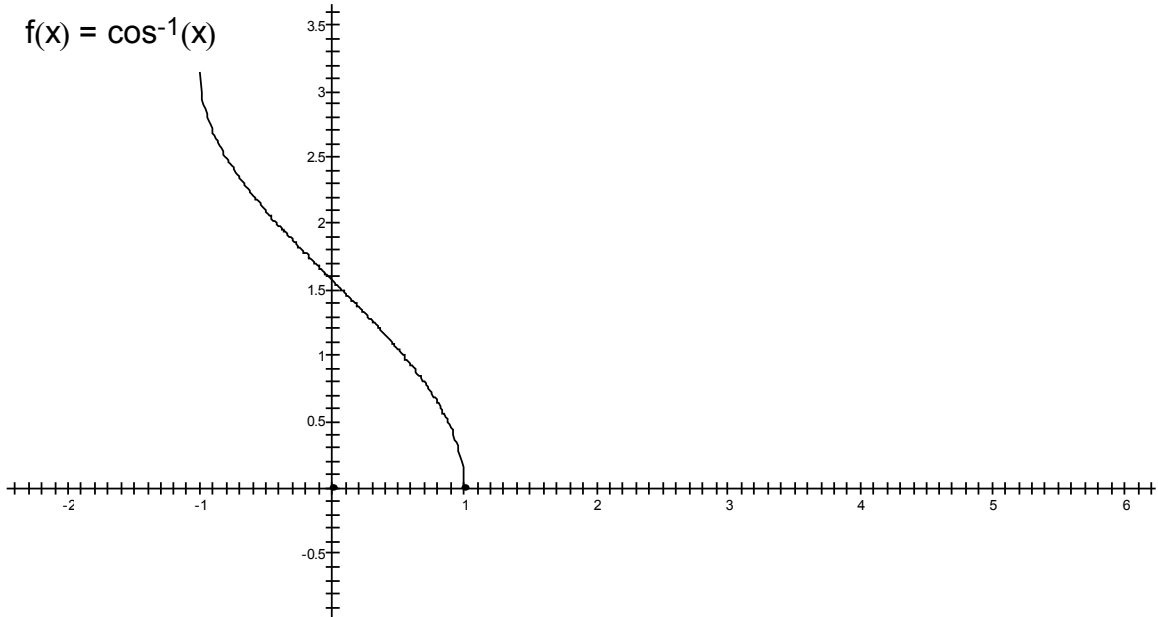Arccsc(x)

The Cosine's domain must be truncated a little differently to get an inverse.

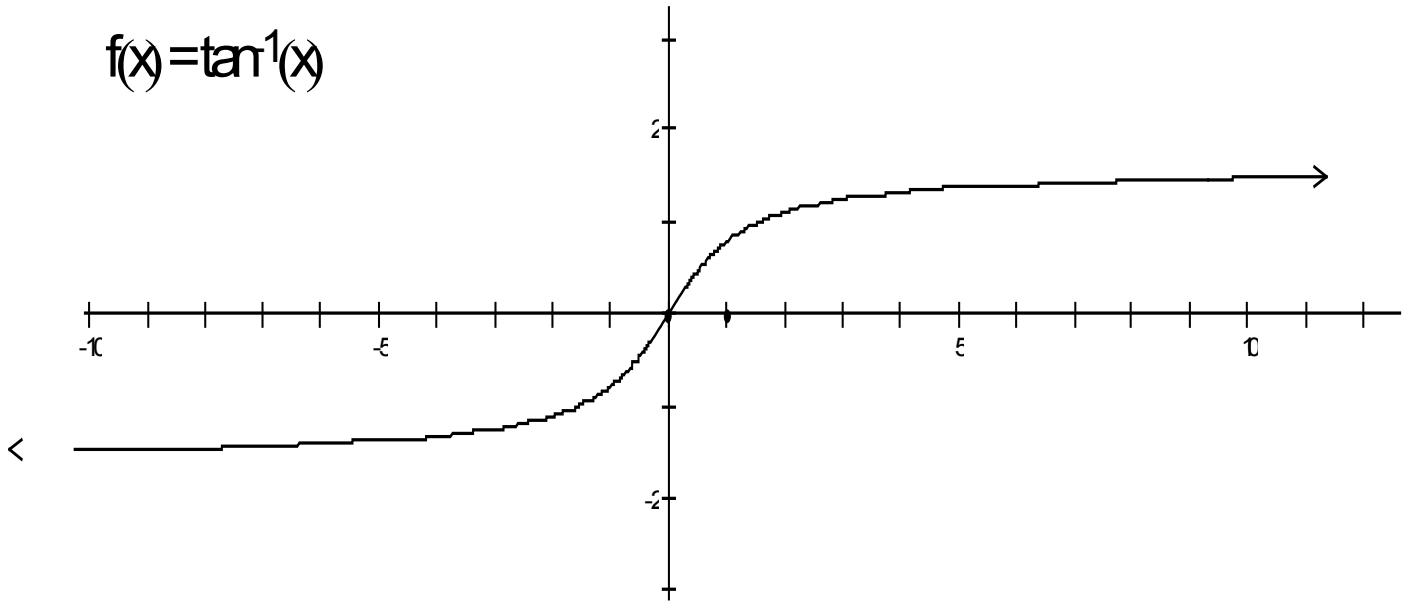Domain $=[0, \pi]$ gives us the inverse cosine as follows:

f(x) = cos^{-1}(x)

Note that it's Domain is [-1,1] like the arcsine, but it's range is $[0,\pi]$

The tangent function has a period of just π so we restrict the Domain to be $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

Note that this is an open interval. Why? The end points are undefined.

The range however is $\left(-\infty,+\infty\right)$

$$f(x)=\tan^{-1}(x)$$

So for $\tan^{-1}(x)$ the domain is $\left(-\infty,+\infty\right)$ but the range is $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$

Similarly we restrict the domain of $ctn(\theta)$ to $(0,\pi)$ with a range of $\left(-\infty,+\infty\right)$ so that

$ctn^{-1}(x)$ has a domain of $\left(-\infty,+\infty\right)$ and a range of $(0,\pi)$.

For $\csc(\theta)$ we restrict the domain to $(-\pi,\pi)$ the same interval as for $\sin(\theta)$ except that it is an open interval.

For $\sec(\theta)$ we restrict the domain to $(0,2\pi)$ the same interval as for $\cos(\theta)$ except that it is an open interval.

The functions described can help you find the angle whose sine, cosine, or tangent is a specific value.

However you should be aware that this angle is not unique.

Example:

Find all angles whose sine is $\dfrac{1}{\sqrt{2}}$

Since $\sin\left(\dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{2}}$

$\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{\pi}{4}$

However the Sine is positive in both the first and 2nd quadrants, so we also have

$\sin^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \dfrac{3\pi}{4}$

But since the sine is periodic we have multiple solutions:

$\left\{\dfrac{\pi}{4} + 2\pi n, \dfrac{3\pi}{4} + 2\pi n\right\} where\, n \in \{...,-2,-1,0,1,2,...\}$

Find all angles where $\tan(\theta) = -1$

We know that $\tan\tan\left(\dfrac{3\pi}{4}\right) = -1$ so $\theta = \dfrac{3\pi}{4}$

But the tangent function is also negative in the fourth quadrant so

$$\theta = \dfrac{3\pi}{4} + \pi = \dfrac{7\pi}{4}$$

But since the tangent function is periodic with period π

$$\theta = \dfrac{3\pi}{4} + \pi n$$

Find all angles where $\csc(\theta) = 3.5$

This is a somewhat trickier problem. We know that the cosecant, like the sine is positive in the first and fourth quadrants. We need a calculator value for this, but there is no csc button on the calculator. We proceed as follows:

$\theta = \csc^{-1} x$

$\csc\theta = x$

$\dfrac{1}{\sin\theta} = x$

$\sin\theta = \dfrac{1}{x}$

$\theta = \sin^{-1}\dfrac{1}{x}$

$\csc^{-1} x = \sin^{-1}\dfrac{1}{x}$

Similarly

$\sec^{-1} x = \cos^{-1}\dfrac{1}{x}$

$\cot^{-1} x = \tan^{-1}\dfrac{1}{x}$

Back to our problem

$$\csc^{-1}(3.5) = \sin^{-1}\left(\frac{1}{3.5}\right) \approx 16.6°$$

In the forth quadrant this will be $-16.6°$

So

$$\theta = 16.6 + 360n \; or -16.6 + 360n$$

More problems:

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) =$$

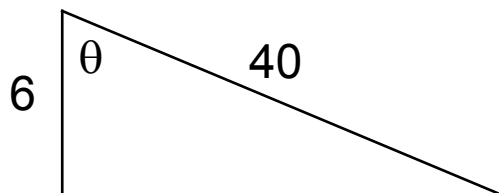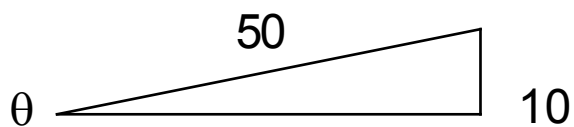$$\cos^{-1}\left(-\frac{1}{2}\right) =$$

$$\tan^{-1}(1) =$$

$$\sin^{-1}(.71) =$$

$$\sin^{-1}(2) =$$

$$\cos^{-1}(2) =$$

Solving some triangles:

HW: 5.5 3-9, 13, 19, 35
HW: 6.4 5-8, 17-19, 23,24, 42