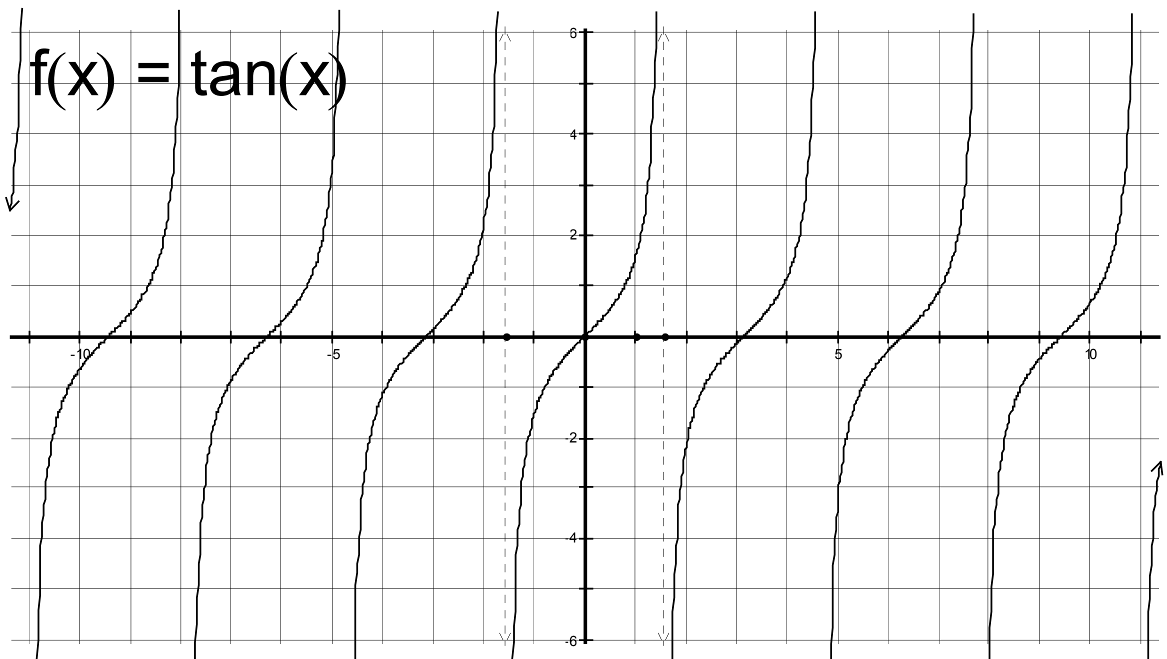
Trigonometry 7 Mathematics 108

More on Trig Functions

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

Let's look at a graph of this function:



The two dashed lines are at $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. What happens to the function at that x value?

These vertical lines are asymptotes.

Where is this function not defined? ___________________

What is the functions period? _________________

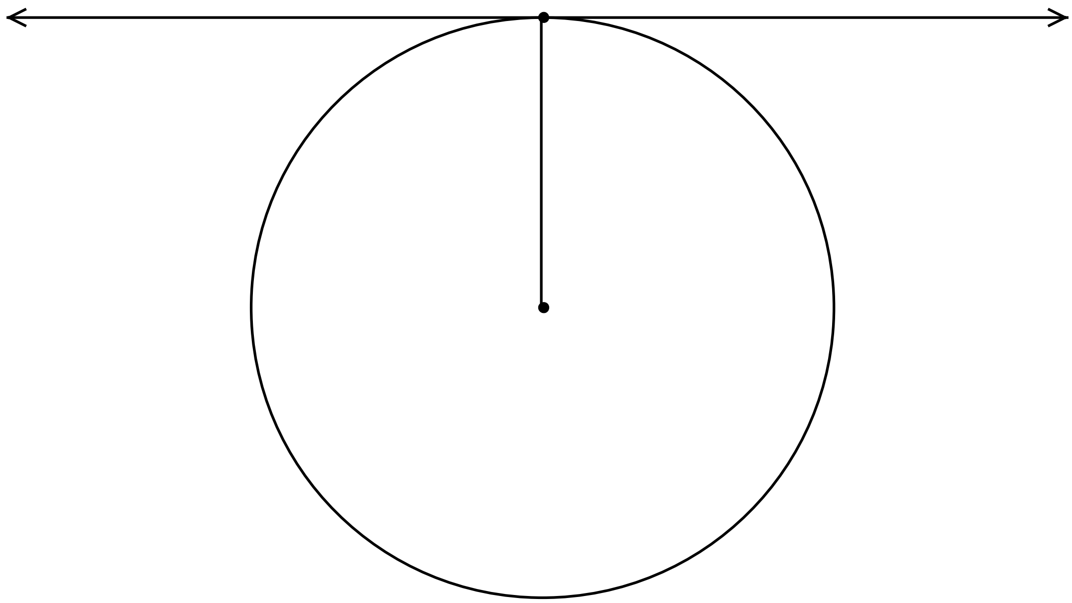
What is the functions Domain: _________________

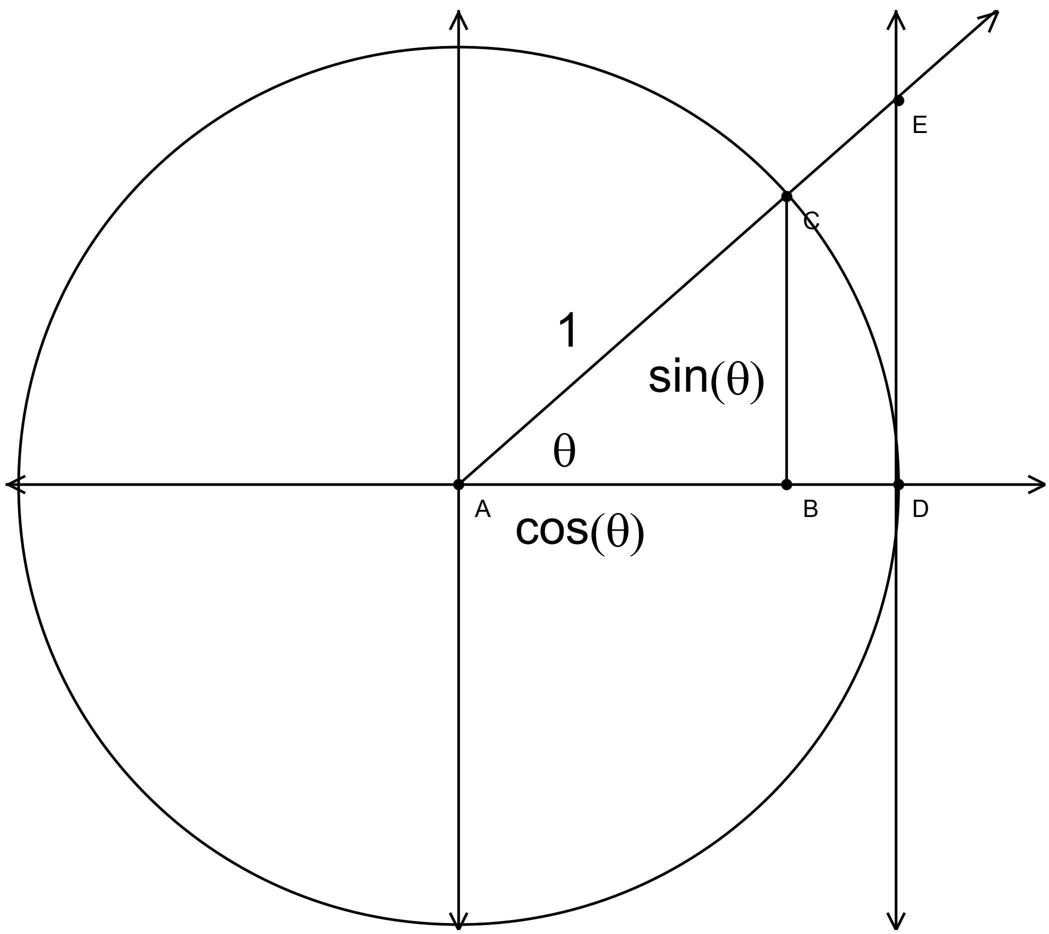
What is the functions Range: _________________

What is the functions Amplitude? _________________(trick question)

Why is the tangent function called **tangent**?

What does the function have to do with a tangent to a circle?





In this diagram, note that since $AC = 1$, then $AB = \cos(\theta)$ and $BC = \sin(\theta)$.

Triangle ABC and ADE are similar, so

$$\frac{BC}{AB} = \frac{DE}{AD}$$

But AD is a radius so $AD = 1$

Plugging in we get

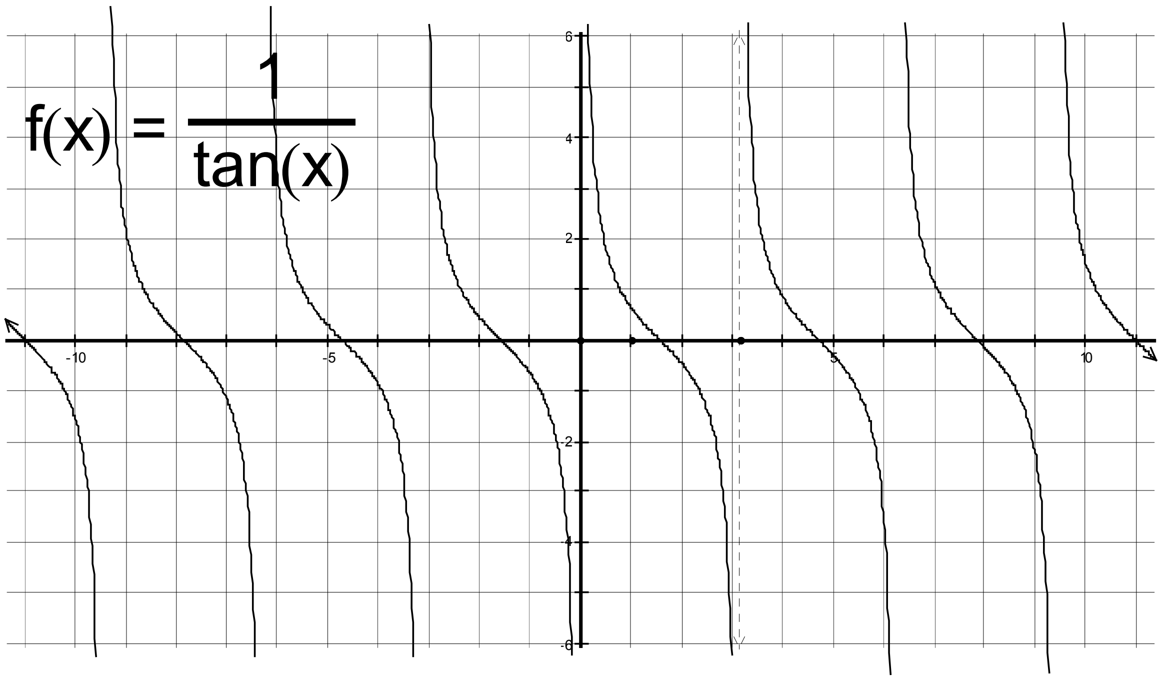
$$\frac{\sin(\theta)}{\cos(\theta)} = \frac{DE}{1}$$

So the length of segment DE is the $\tan(\theta)$

Well DE is a segment tangent to the circle!

$$\cot(x) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$$

Let's look at a graph of this function:



How does this compare with the tangent function?

Where is this function not defined? ___________________

Where are the vertical asymptotes

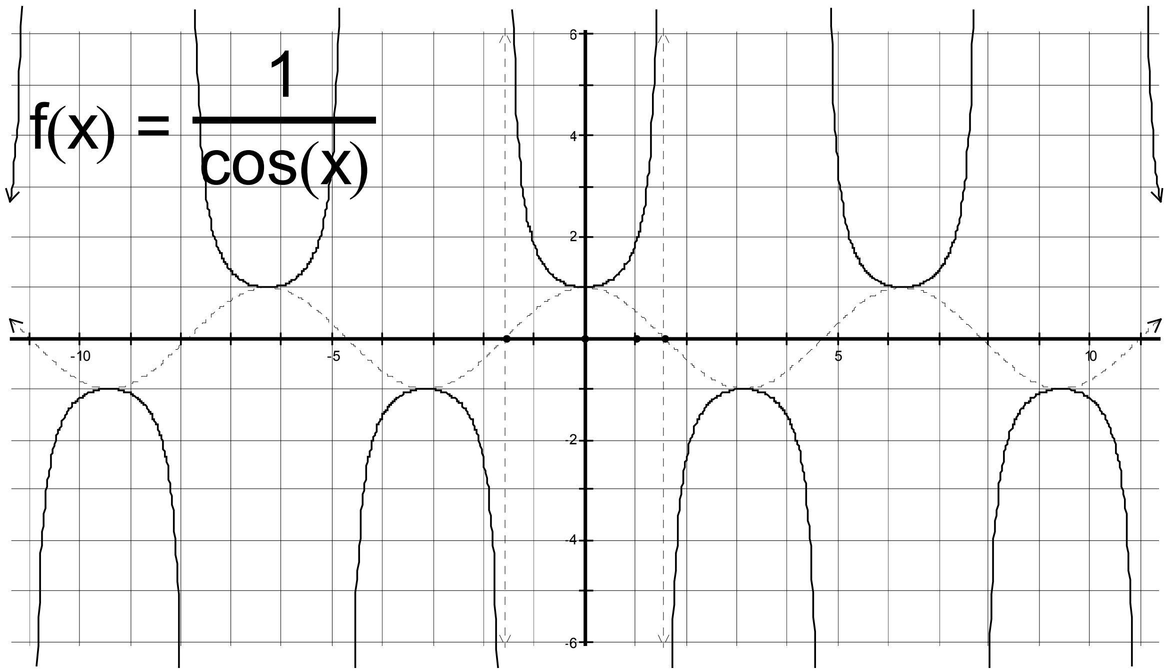
What is the functions period? _________________

What is the functions Domain: _________________

What is the functions Range: _________________

$$\sec(x) = \frac{1}{\cos(x)}$$

Let's look at a graph of this function:



How does this compare with the cosine function?

Where is this function not defined? ___________________

Where are the vertical asymptotes

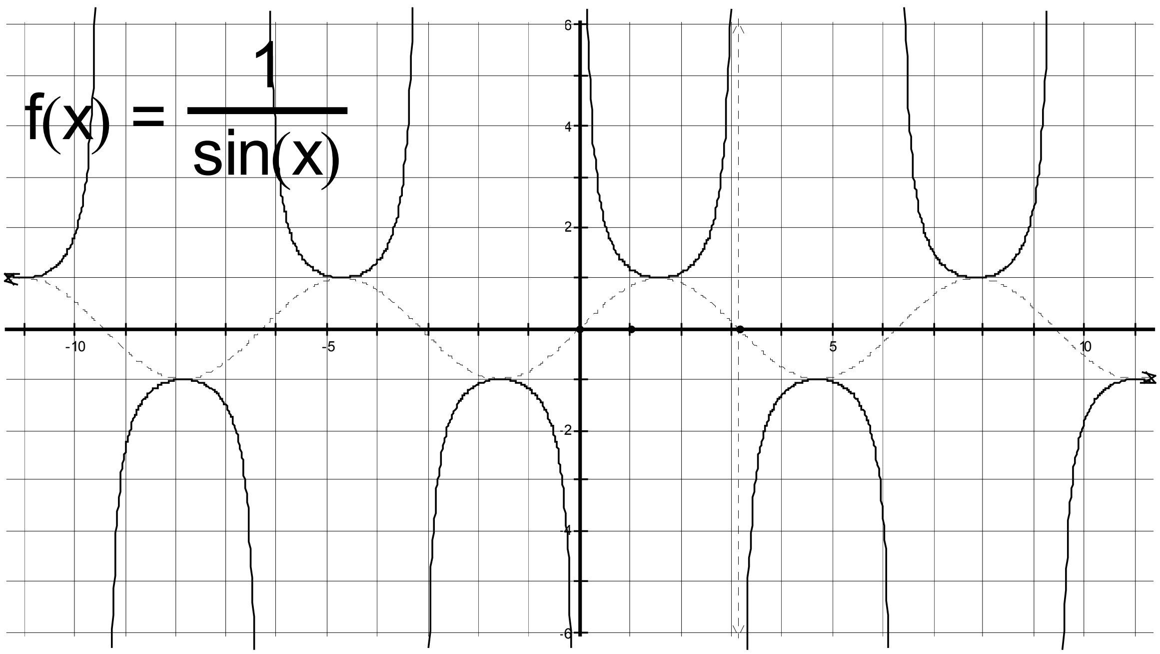
What is the functions period? _________________

What is the functions Domain: _________________

What is the functions Range: _________________

$$\csc(x) = \frac{1}{\sin(x)}$$

Let's look at a graph of this function:



How does this compare with the sine function?

Where is this function not defined? ___________________

Where are the vertical asymptotes

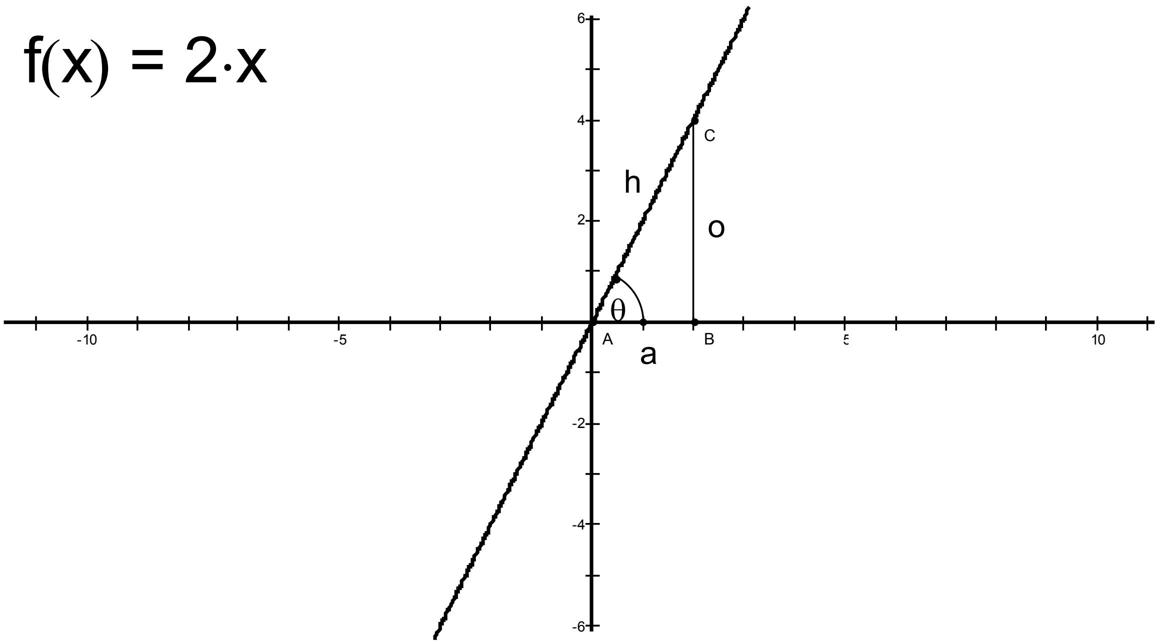
What is the functions period? _________________

What is the functions Domain: _________________

What is the functions Range: _________________

A special property of the tangent function:

Take a linear equation going through the origin (0,0)



Note that:

$$\sin(\theta) = \frac{o}{h}$$

$$\cos(\theta) = \frac{a}{h}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{o}{h}}{\frac{a}{h}} = \frac{o}{a}$$

But

$$\frac{\Delta y}{\Delta x} = \frac{o}{a} = \tan(\theta)$$

So the tangent function gives us the slope of a line!

Graphing the other Trigonometric functions:

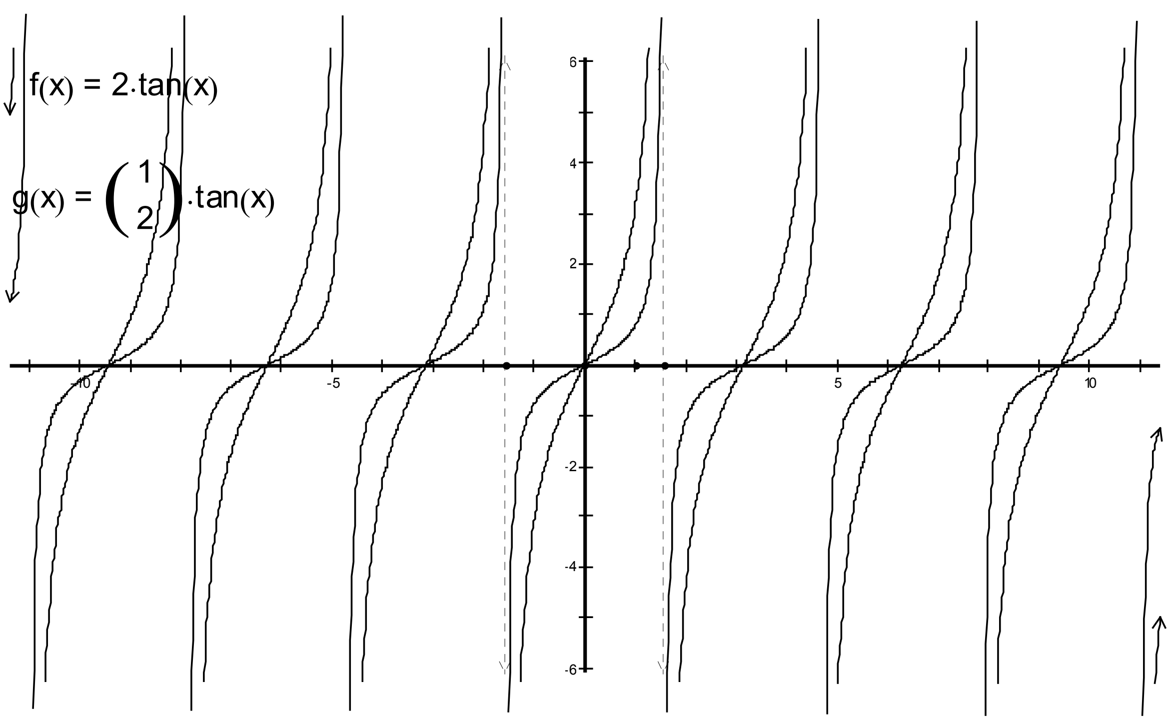
Example: $f(x) = 2\tan\left(2\left(x - \frac{\pi}{2}\right)\right) + 3$

D=3 still is a vertical shift up

C=$\frac{\pi}{2}$ is still a horizontal shift to the right

B = 2 still affects the period in the same way $P = \frac{\pi}{|B|}$

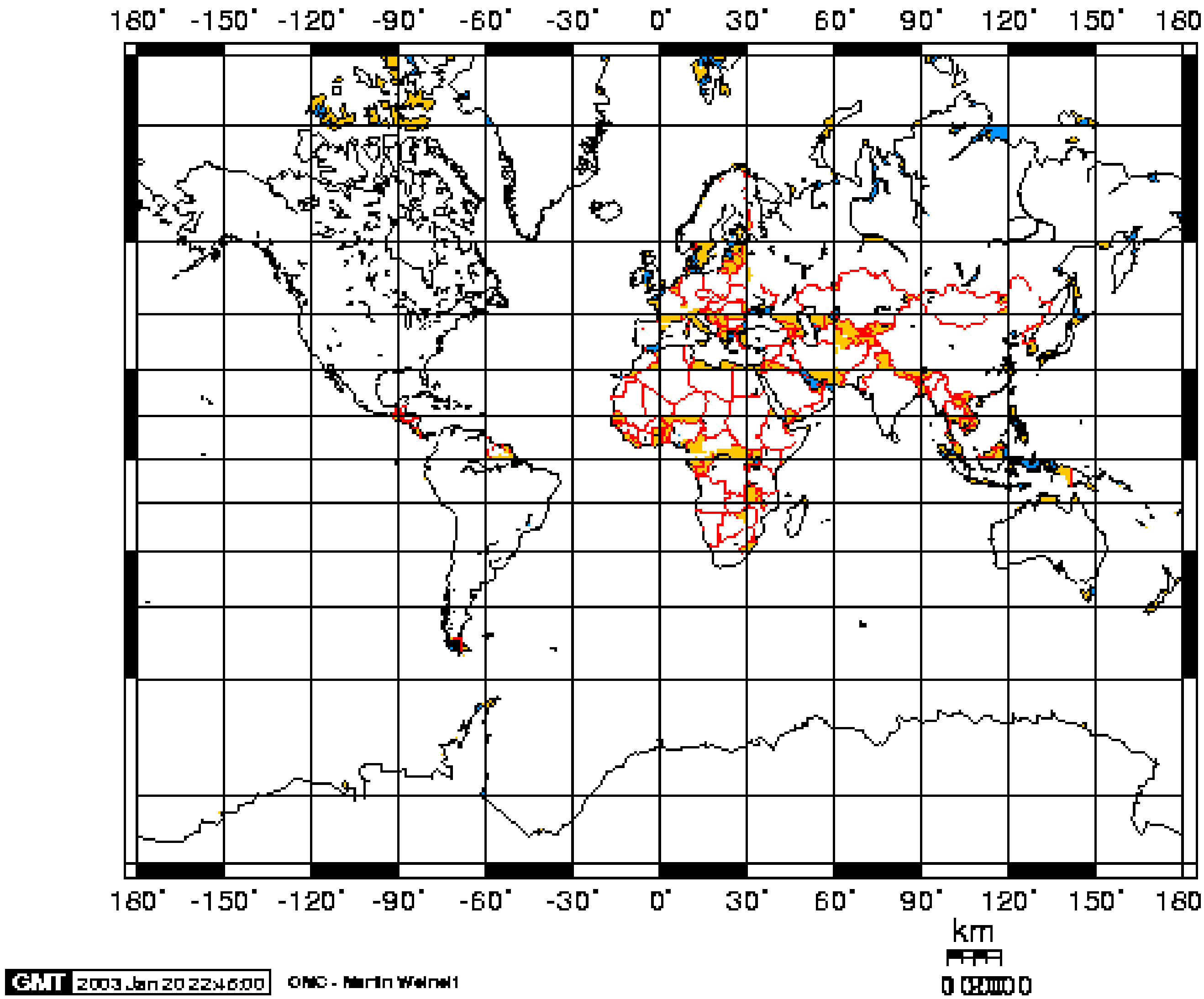
How about A?



Class HandOut

Curious property of tangent used in a Mercator projection.

Note, this distorted projection is renowned for causing confusion about the size of the contents. It looks like Greenland is almost the size of Africa. In reality it is much smaller. Also, Alaska looks half the area of the US. Alaska is big, but not nearly that big.



Note that the latitude lines are mapped tan(θ) from the center line. That means that the North and South Pole cannot be shown because they are at infinity.

The importance of this projection of a sphere onto a flat surface is that it preserves angles. That means that if you draw a straight line on the map, it really is a straight line or great circle on the map.

HW: 5.4: 5, 7, 9, 18, 19, 24,49, 56