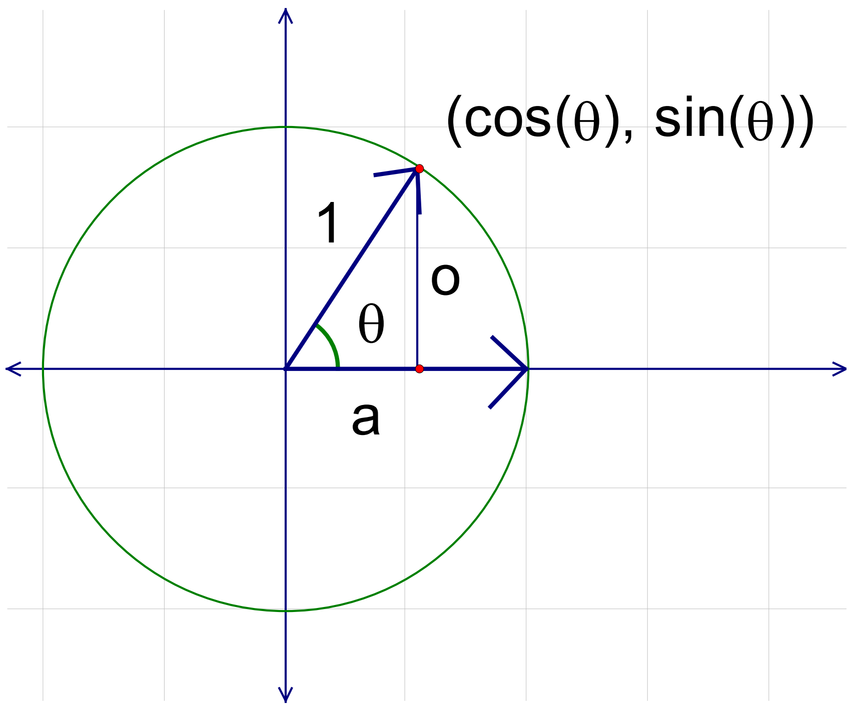
The Unit Circle

Unit Circle View of Trig functions

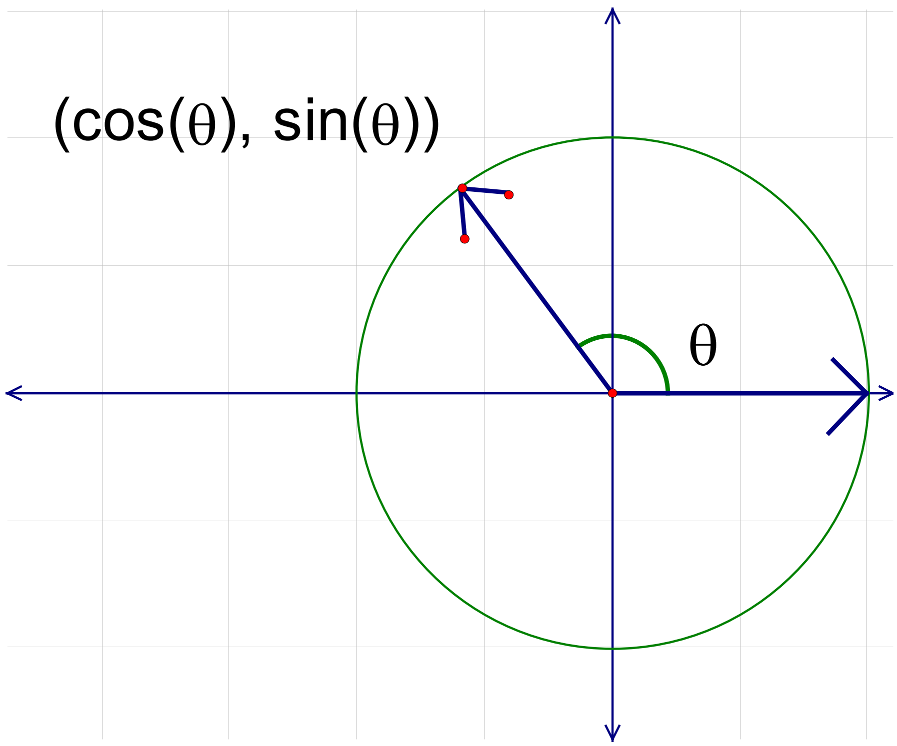
Here we redefine the sin and cosine functions as coordinates on the unit circle.



Take a look at this animation and notice that the sine and cosine function are doing the same thing, only out of sync by 90°

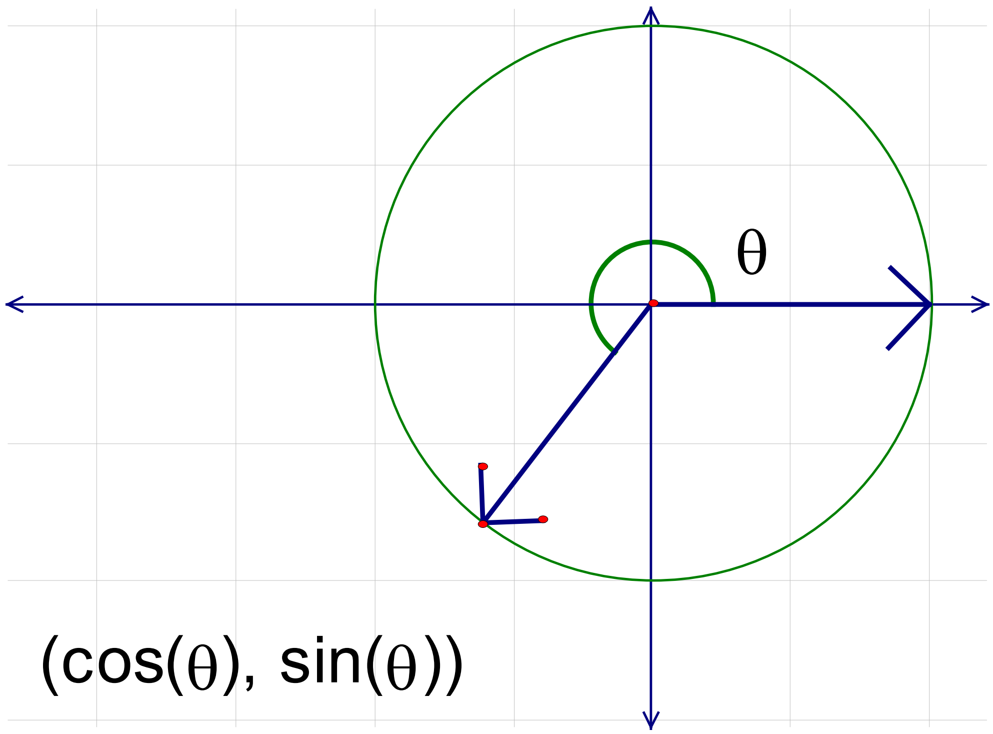
http://schoenbrun.com/foothill/math48c-2/gsps/CircularMotion.gsp

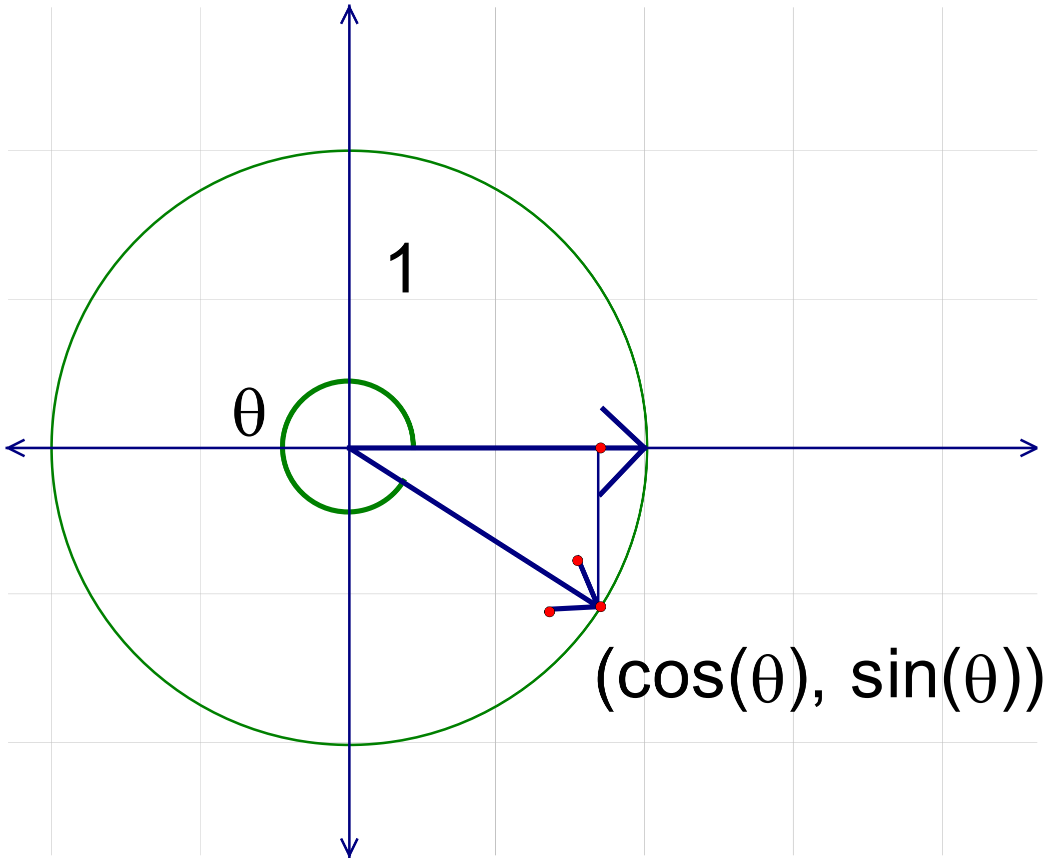
Note that θ can have any real number as its value.



In the 2nd quadrant the sine is still positive but the cosine is negative.

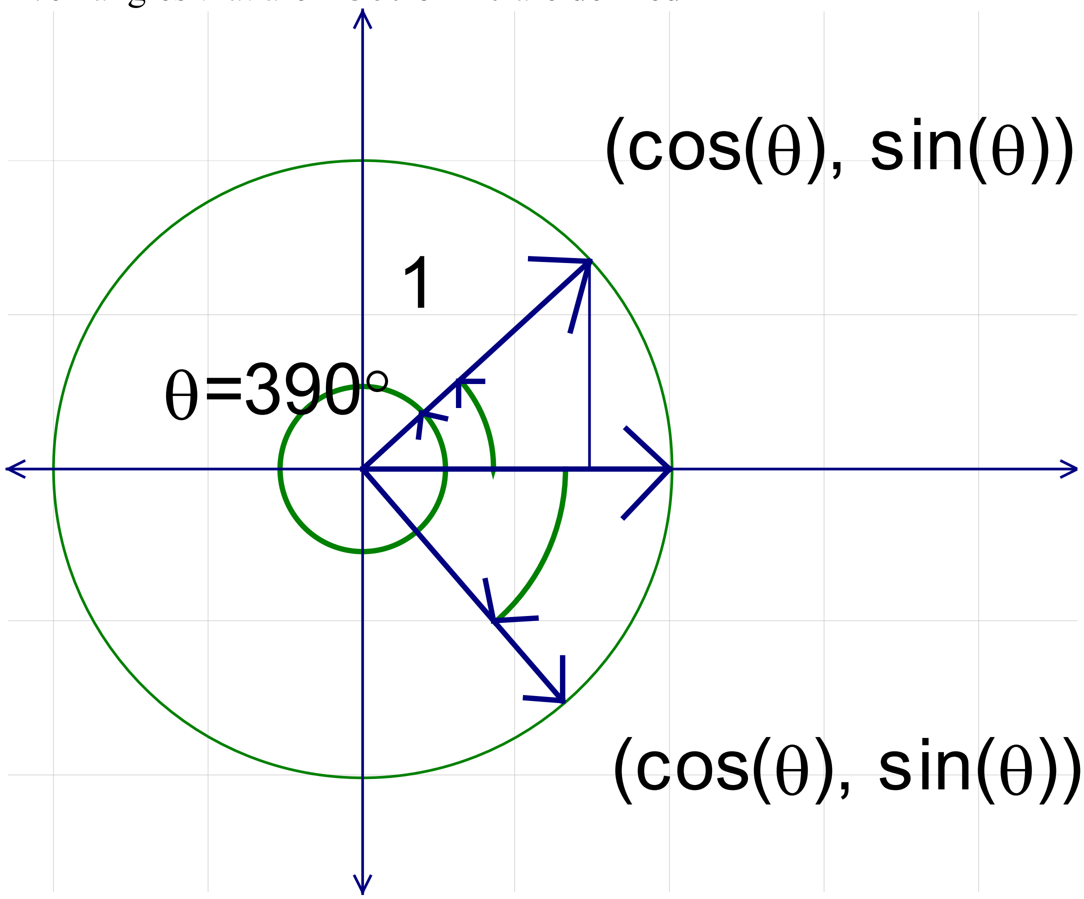
In the third quadrant both are negative



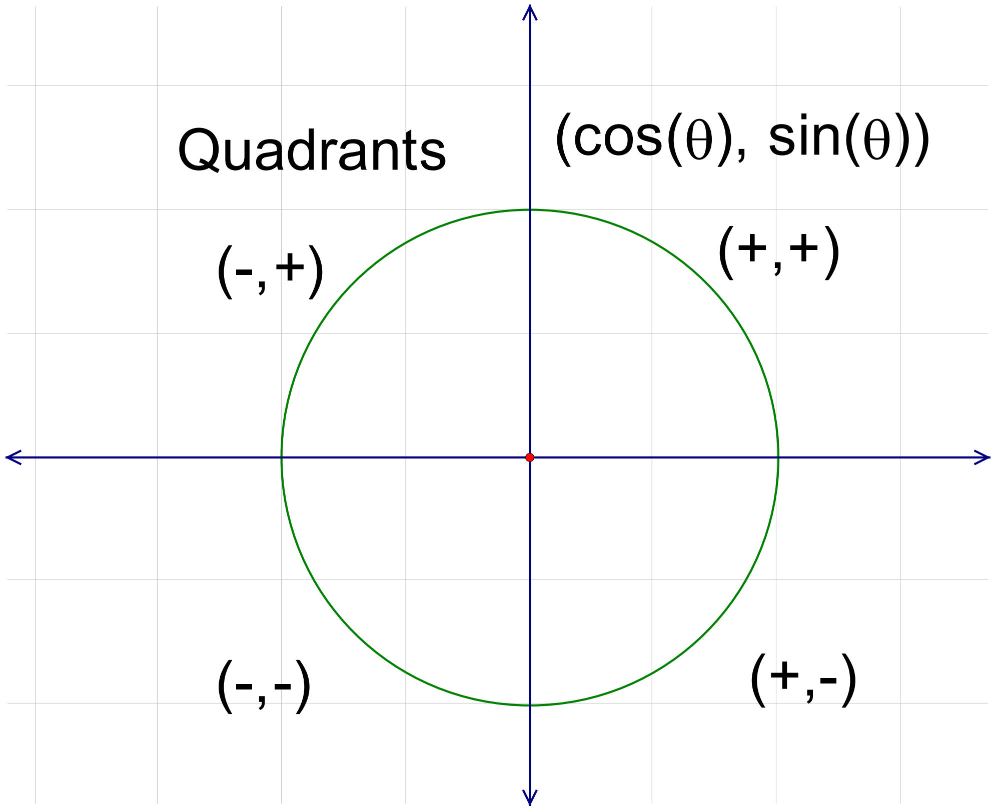


And in the 4th quadrant only the sine is negative.

Even angles that are > 360 or < 0 are defined



Here we have a map of showing what sign's of the two functions in each quadrant.



Hint: It is a good idea to become familiar with these values.

There are the sine and cosine value of some important angles:

$$\sin\left(0^\circ\right) = 0$$

$$\cos\left(0^\circ\right) = 1$$

$$\sin\left(90^\circ\right) = 1$$

$$\cos\left(90^\circ\right) = 0$$

$$\sin\left(180^\circ\right) = 0$$

$$\cos\left(180^\circ\right) = -1$$

$$\sin\left(270^\circ\right) = -1$$

$$\cos\left(270^\circ\right) = 0$$

Reference Angles

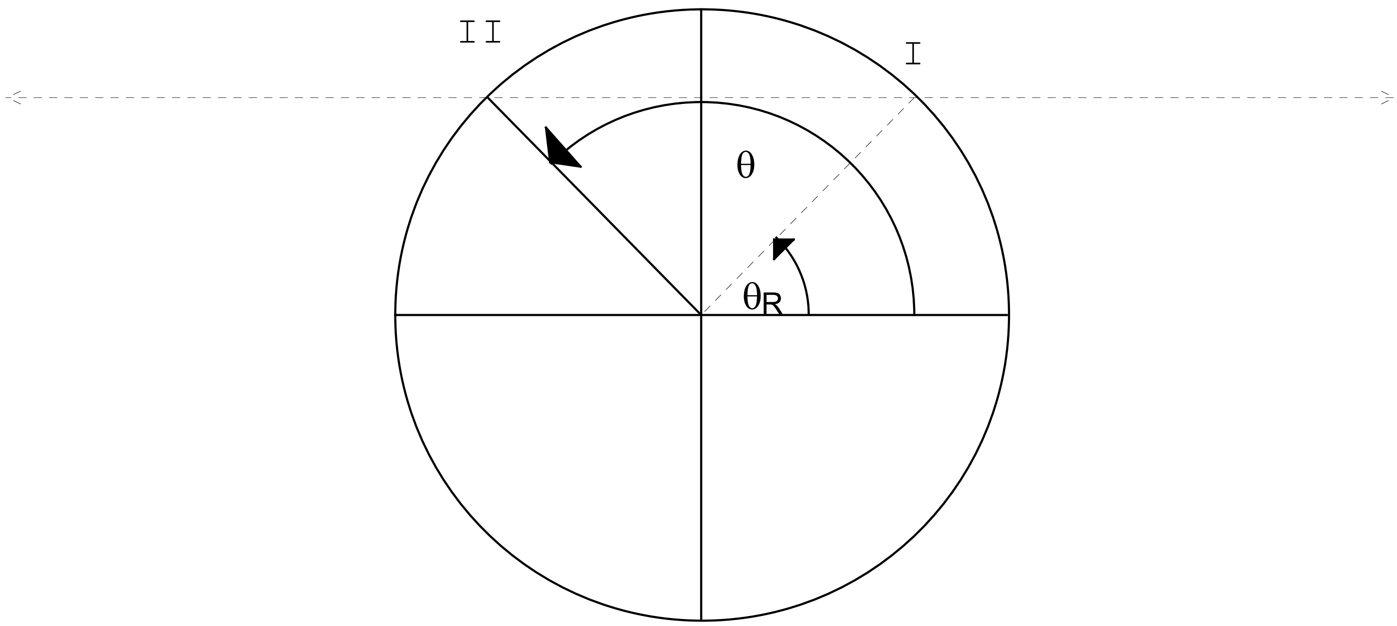
A **Reference angle** is an angle in the first quadrant.
Every angle will have a corresponding References Angle.

The trigonometric functions of an angle will have the same value as for the corresponding reference angle or its negative.

First we look at angles $0^\circ \leq \theta \leq 360^\circ$

Note, that the reference angle for any angle in the first quadrant is itself.

For an angle in the 2nd quadrant, draw a line parallel to the X-axis through the point where the angle intersects the unit circle and then find where this line intersects the unit circle in the first quadrant.



Note that for the 2nd quadrant $\theta_R = 180^\circ - \theta$

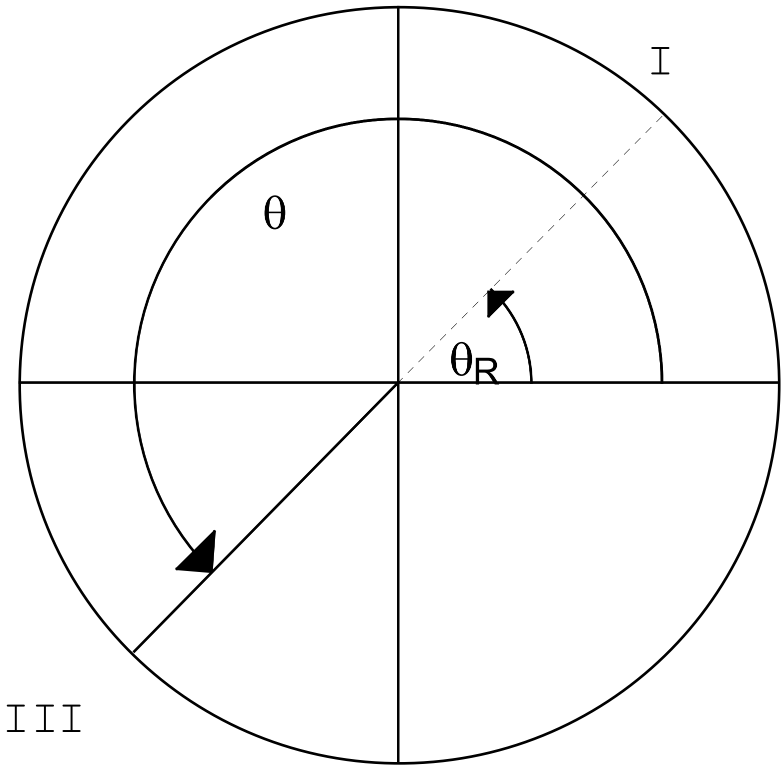
Also note that

$\sin(\theta_R) = \sin(\theta)$

but

$\cos(\theta_R) = -\cos(\theta)$

For an angle in the 3rd quadrant, extend the terminal ray in the opposite direction and find where it intersects the unit circle.



Note that for the 3rd quadrant $\theta_R = \theta - 180^\circ$

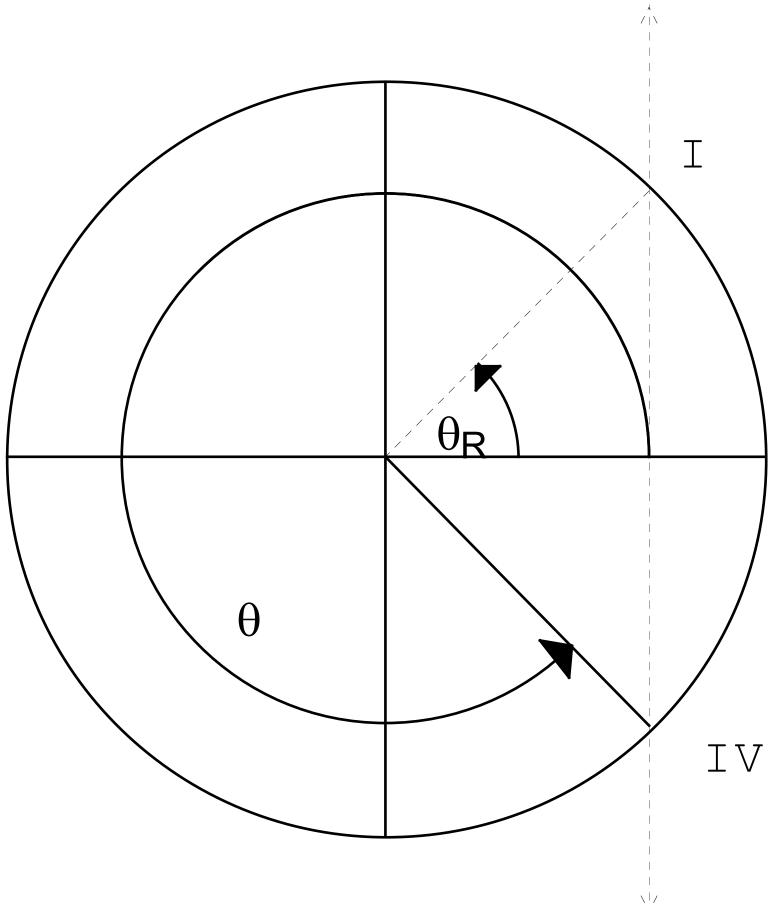
Also note that

$\sin(\theta_R) = -\sin(\theta)$

and

$\cos(\theta_R) = -\cos(\theta)$

Finally for an angle in the 4th quadrant draw a line parallel to the y axis from where the angle intersects the unit circle.



Note that for the 4th quadrant $\theta_R = 360^\circ - \theta$

Also note that

$\sin(\theta_R) = -\sin(\theta)$

but

$\cos(\theta_R) = \cos(\theta)$

For any angle $\theta < 0^\circ$ or $\theta > 360^\circ$

There is some angle θ_u, u as in unit-circle for which

$$\theta = \theta_u + n360^\circ$$

where n is an integer such that $0^\circ \le \theta_u \le 360^\circ$

With $\sin(\theta_u) = \sin(\theta)$

and

$$\cos(\theta_u) = \cos(\theta)$$

Recall that:

$$\sin(90 - \theta) = \cos(\theta)$$
$$\cos(90 - \theta) = \sin(\theta)$$

For one of these functions where $45^\circ < \theta \le 90^\circ$, you can find the value using the complementary function.

So for calculation purposes, we only need to know the values of the sine and cosine between 0° and 45°.

The exact values of the sine and cosine can be determined exactly for angles that multiples of 30° and 45°.

Functions: We can find an exact expression for any multiple of 30° or 45°

Angle	Sine	Cos	Angle	Sine	Cos	Angle	Sine	Cosine	Angle	Sine	Cos
0°			90°			180°			360°		
30°			120°			210°			300°		
45°			135°			225°			315°		
60°			150°			240°			330°		

Solutions

Angle	(cos,sin)	Angle	(cos,sin)	Angle	(cos,sin)	Angle	(cos,sin)
0°	(+1,0)	90°	(0,+1)	180°	(-1,0)	360°	(0,-1)
30°	$\left(+\frac{\sqrt{3}}{2},+\frac{1}{2}\right)$	120°	$\left(-\frac{1}{2},+\frac{\sqrt{3}}{2}\right)$	210°	$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	300°	$\left(+\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
45°	$\left(+\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}\right)$	135°	$\left(-\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}\right)$	225°	$\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$	315°	$\left(+\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$
60°	$\left(+\frac{1}{2},+\frac{\sqrt{3}}{2}\right)$	150°	$\left(-\frac{\sqrt{3}}{2},+\frac{1}{2}\right)$	240°	$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	330°	$\left(+\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$

HW: 5.2: 5-10, 13-15, 23-26