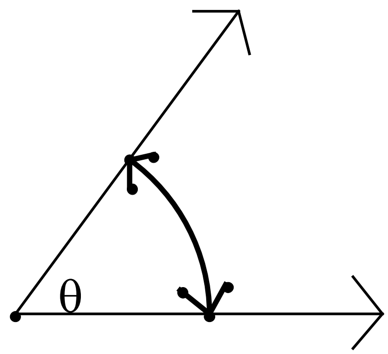
Trigonometry 3 Mathematics 108

Angle Measure (6.1)

An angle is defined by two rays (or line segments) that share an endpoint.
We define one of the rays of an angle the **initial side**, and the other ray the **terminal side.**



Angle Measure Units

Traditionally we use **degrees** as the unit of measure of an angle. We break up a complete circle into 360 degrees and write it:

$360°$

Where does this 360 come from?
An early civilization, the Babylonians used a base 60 system.
60 seconds in a minute
60 minutes in an hour.

60 has the nice property that you can divide it by 2, 3, 4, 5, and 6.

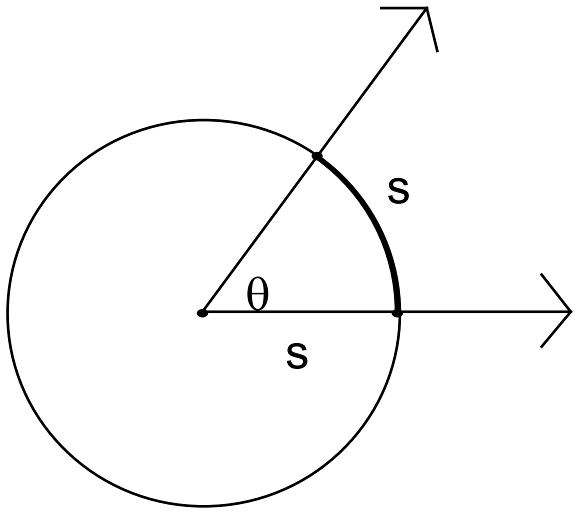
This is not the only set of units that are used for angles.

Also used is a grad which divides a circle into 400 parts.

We are going to use a more **natural** unit called a **Radian**.
Using this unit divide a circle into 2π Radians.
This sounds a bit strange, $2\pi = 6.28....$

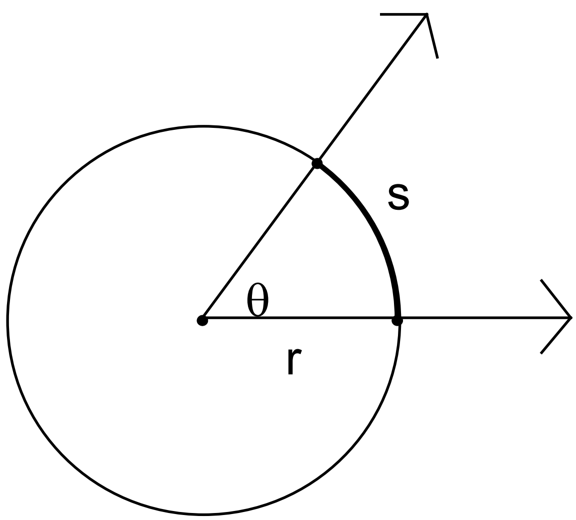
You might wonder about me calling it a natural unit.
Recall that Natural logarithms use the base e=2.71828... also a strange number.

Definition of Radian measure
If a circle of radius 1 is drawn with the vertex of an angle at its center, then the measure of this angle in radians is the length of the arc that subtends the angle.



Radians = s

Alternative definition
If a circle of radius r is drawn with the vertex of an angle at its center, then the measure of this angle in radians is the length of the arc that subtends the angle, divided by r.



Radians = s/r

Why are these definitions equivalent?

From geometry we know that all circles are similar. So s/r is the same for all circles with angle θ

Note that we always have $$s = r\theta$$

Examples:

(a) Find the length of an arc of a circle with radius 10 m that subtends a central angle of 30°?

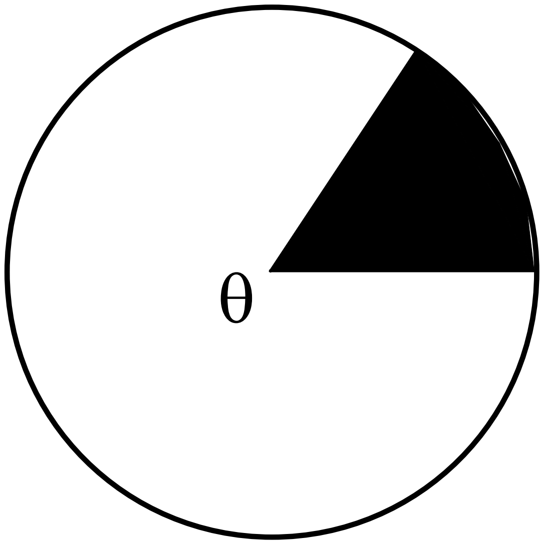
$$30^{\circ} = \frac{\pi}{6} Radians$$

$$10 meters \times \frac{\pi}{6} Radians = \frac{10\pi}{6} meters \approx 5.23 meters$$

(b) A central angle θ in a circle of radius 4 m is subtended by and arc of length 6 m. Find the measure θ of in radians.

$$\frac{6 meters}{4 meters} = 1.5 Radians = 1.5 Radians$$

Finding the area of a circular sector



We know that the area of a circle is $A = \pi r^2$ and that the number of radians in that circle are $\theta = 2\pi$.

So the area of the sector is $A = \pi r^2 \dfrac{\theta}{2\pi} = \dfrac{1}{2} r^2 \theta$

Example:

Find the area of a circle with central angle 60° if the radius of the circle is 3 m.

$$60^\circ = \frac{\pi}{3} \, Radians$$

$$A = \frac{1}{2}(3m)^2 \frac{\pi}{3} = \frac{3\pi}{2} m^2 \approx 14m^2$$

Conversion between Degrees and Radians

Tool: http://www.schoenbrun.com/foothill/math48c-2/gsps/Angle1.gsp

Let's look at various values.

1 radian ≈ 57.296 degrees

1 degree $\approx .01745$ radians

A table of useful values

Degrees	Radians
360°	2π
180°	π
90°	$\frac{\pi}{2}$
60°	$\frac{\pi}{3}$
45°	$\frac{\pi}{4}$
30°	$\frac{\pi}{6}$

You are going to have to convert between radians and degrees a lot in this course. You could try to remember a formula. I think an easier way is to remember something simpler:

$360° = 2\pi$ Radians.

Using **unit-analysis** we can see immediately that

$$radians = \frac{360^\circ}{2\pi} degrees$$
$$degrees = \frac{2\pi}{360^\circ} radians$$

Then you can remove the extra factor of 2 if you like getting the usual formulae:

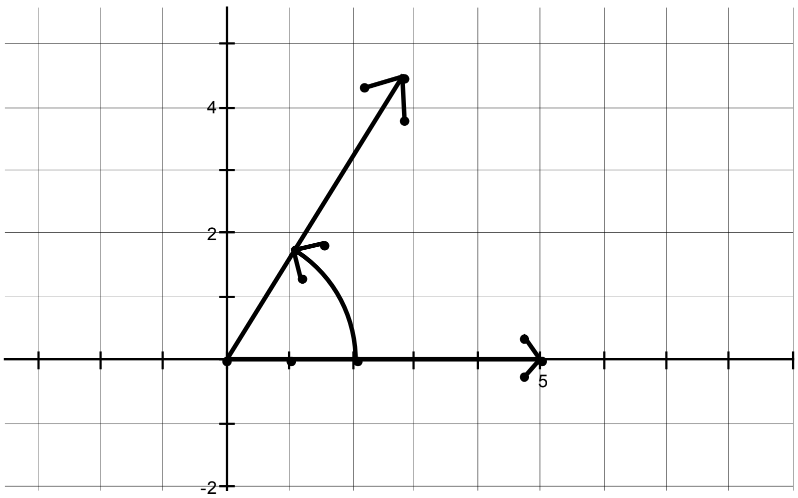
$$radians = \frac{180^\circ}{\pi} degrees$$
$$degrees = \frac{\pi}{180^\circ} radians$$

Let's take a minute to look at a short review of this information:

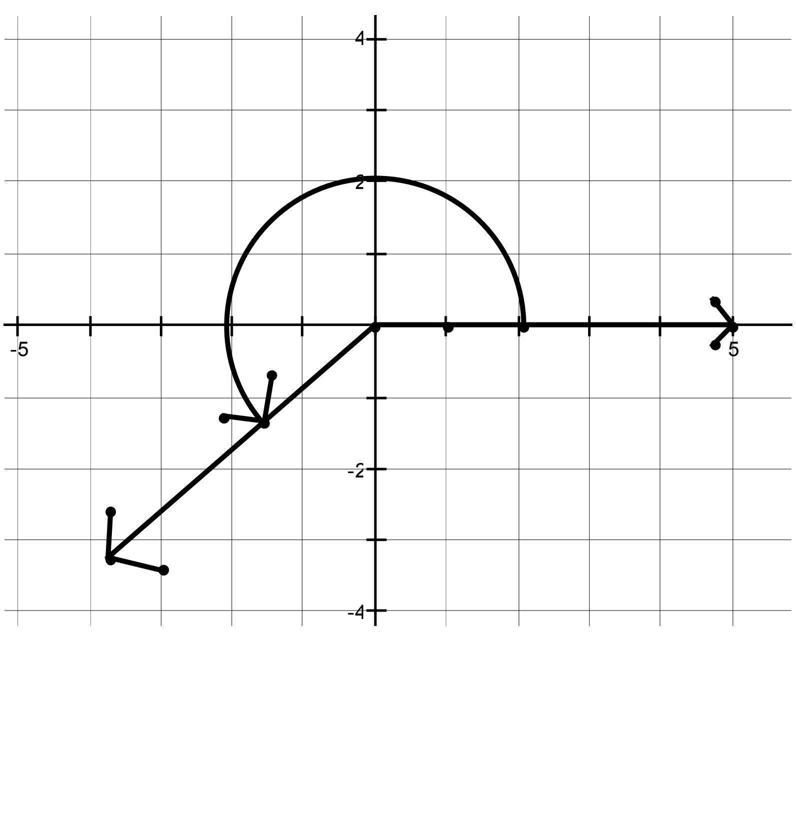
Video:http://www.schoenbrun.com/foothill/math48c-2/mpeg/Radians-1.33.mpg

Angles in standard position

First we define an angle to be in standard position, if vertex of the angles rays are on the origin, and the initial side is on the *X*-axis.



Or



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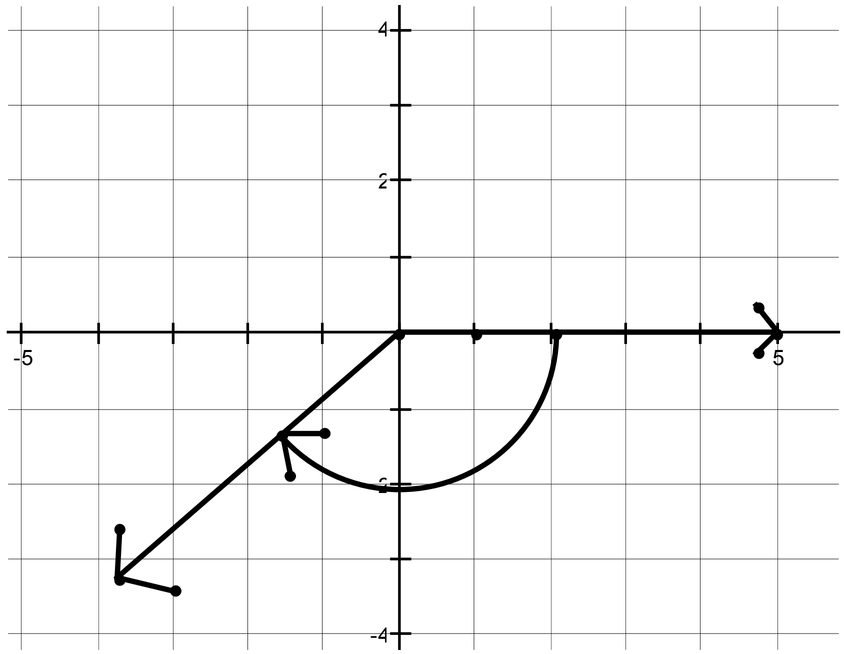

Note that if the direction is counter clock wise, we give the angle a positive direction.
If the direction is clock wise, we give the angle a negative direction.

An angle can equal or exceed 360°s or 2π by wrapping around the origin multiple times.

If the two angles have rays that coincide we say they are **co-terminal**.

What is the relationship between two angles α and β are the possible measures of co-terminal angles?

$\alpha = \beta + 360^{\circ} n$ where n is an integer = {...,-2, -1, 0, 1, 2, ...}
or

$\alpha = \beta + 2\pi n$ where n is an integer = {...,-2, -1, 0, 1, 2, ...}

HW 6.1: 5, 8, 9, 10, 14, 16, 17, 18, 19, 20, 21, 26, 41, 48, 57, 65