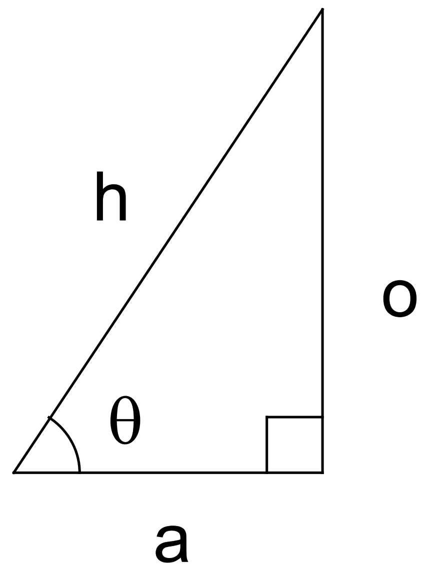
Triangle Trigonometry

View of the trigonometry functions from the point of view of a Right Triangle



h-hypotenuse

o-opposite

a-adjacent

$$\sin(\theta) = \frac{o}{h}$$

$$\cos(\theta) = \frac{a}{h}$$

$$\tan(\theta) = \frac{o}{a}$$

Memory Device: SOH-CAH-TOA (Sounds Indian)

SOH (Sin = O/H)
Sine = Opposite/Hypoteneuse

CAH (Cos=A/H)
Cosine = Adjacent/Hypoteneuse

TOA (Tan=O/A)
Tangent = Opposite/Adjacent

This is only useful when you have a right triangle. Note that $0 < \theta < 90^\circ$.

Review of Sines and Cosines

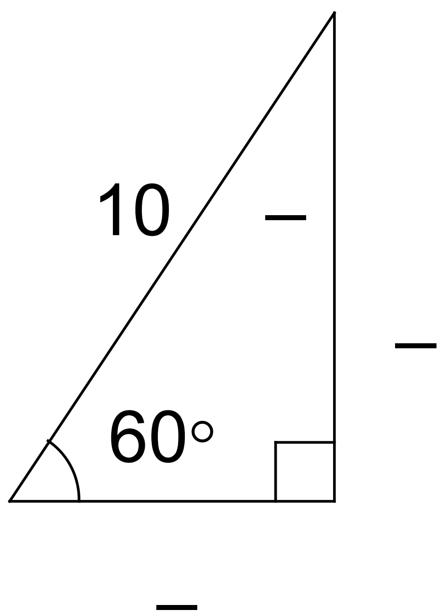
http://schoenbrun.com/foothill/math48c-2/mpeg/Ratios.mp4

What kind of problems can we solve with this?

Given any two of θ, h, a or o, we can find all missing angles and sides of the triangle.

Example: Given a right triangle with hypotenuse length 10 and missing sides
and $\theta = 60°$ what are the missing angles and sides?

Note: This is a 30/60/90 triangle whose side ratio's you should know.
(Show How)



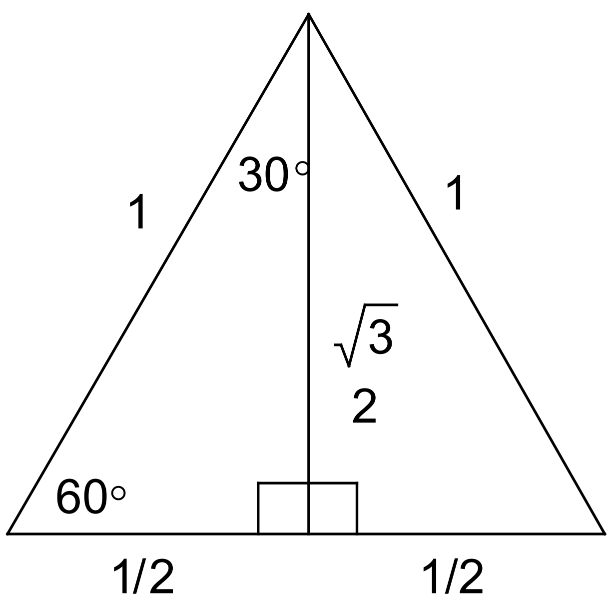
$$\sin(60°) = \sqrt{3}/2$$
$$\cos(60°) = 1/2$$

Also, there are buttons on your calculator for these functions
and their inverses.

Digression - Important Triangles
Special Angles with Exact Values

Using our knowledge of special triangles from geometry:
30/60/90 triangles:

Take an equilateral triangle with sides 1 whose angles must all be 60°.
Drop a perpendicular from it's highest point to the base.
This divides the triangle into two congruent triangles.
By symmetry the angles of each of these triangles must be 30/60/90 degrees.
The base is 1/2 and the hypotenuse is 1 so by the Pythagorean theorem we get the second leg to be $\frac{\sqrt{3}}{2}$



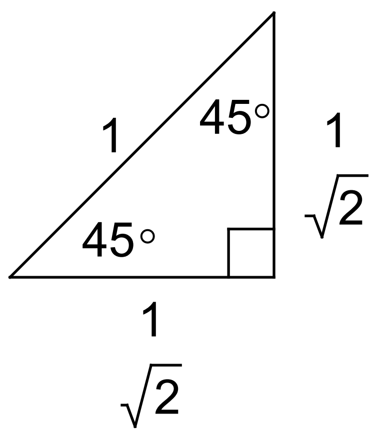
This tells us that

$$\sin\left(60^\circ\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(60^\circ\right) = \frac{1}{2}$$

Isosceles right triangles:

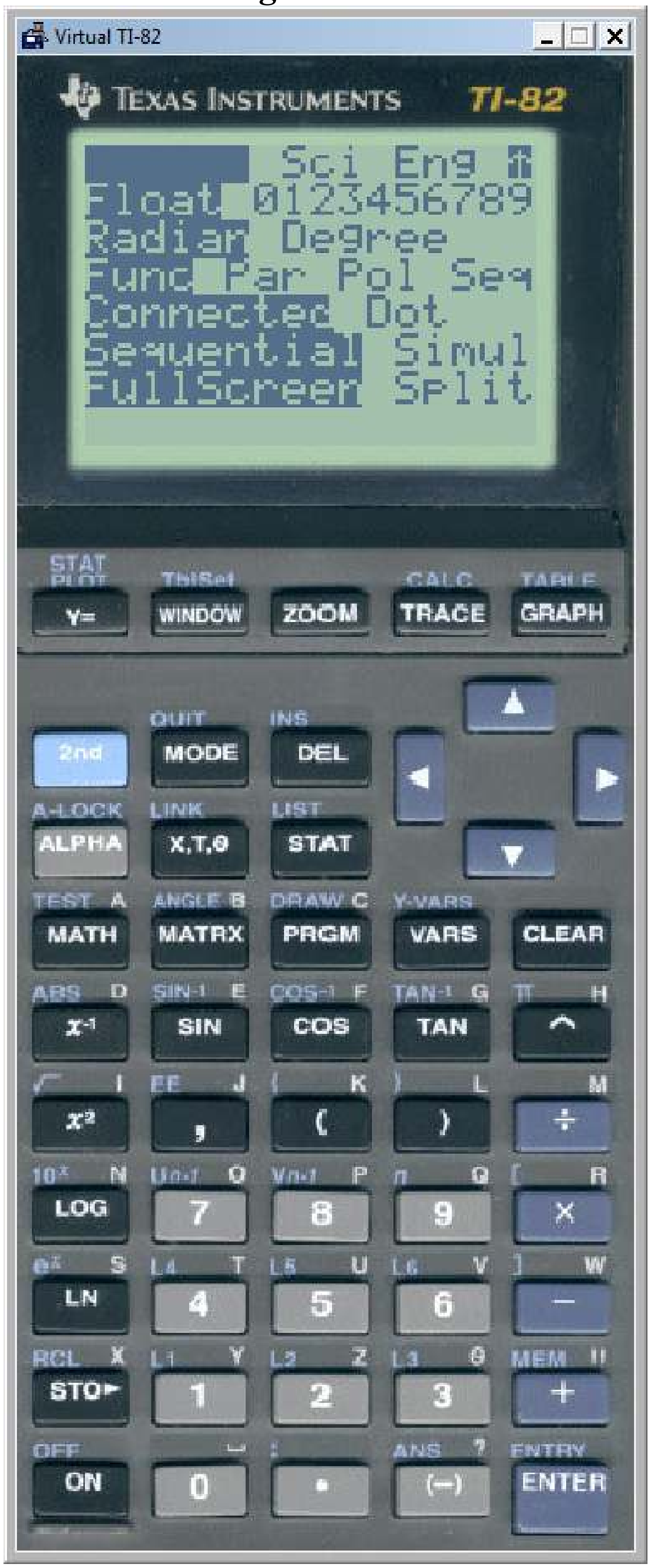
Given a right isosceles triangle with hypotenuse 1 we know immediately that the smaller angles are 45° and by the Pythagorean theorem, the legs are $\frac{1}{\sqrt{2}}$



This tells us that

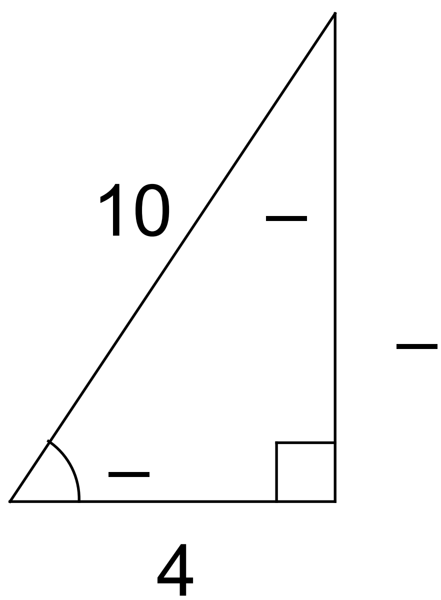
$$\sin\left(45^\circ\right) = \frac{1}{\sqrt{2}}$$
$$\cos\left(45^\circ\right) = \frac{1}{\sqrt{2}}$$

Using a Calculator to find Sines and Cosines
ALWAYS CHECK THE MODE FIRST!!!!!!!
For Now use Degree Mode!



Example:

Given a right triangle with hypotenuse length 10 and leg 4, what are the missing angles and sides?



Since we know

$$\cos(\theta) = \frac{4}{10} = .4$$

Using a scientific calculator we find

$\theta \approx 66.42182^\circ$ using the $\cos^{-1}$ function key

The table below shows that the cosine of .4 is

$66^\circ 25 < \theta < 66^\circ 26$

Converting from degrees+minutes to degrees

$$66^\circ 25 = 66 + \frac{25}{60} = 66.417$$

$$66^\circ 26 = 66 + \frac{26}{60} = 66.433$$

Some ancient history

How we used to get the values of arbitrary trig functions from a table.

NATURAL TRIGONOMETRIC FUNCTIONS TO FIVE PLACES

22° (202°) (337°) 157°

′	Sin	Tan	Cot	Cos	Sec	Csc	′
0	.37461	.40403	2.4751	.92718	1.0785	2.6695	60
1	.37488	.40436	2.4730	.92707	1.0787	2.6675	59
2	.37515	.40470	2.4709	.92697	1.0788	2.6656	58
3	.37542	.40504	2.4689	.92686	1.0789	2.6637	57
4	.37569	.40538	2.4668	.92675	1.0790	2.6618	56
5	.37595	.40572	2.4648	.92664	1.0792	2.6599	55
6	.37622	.40606	2.4627	.92653	1.0793	2.6580	54
7	.37649	.40640	2.4606	.92642	1.0794	2.6561	53
8	.37676	.40674	2.4586	.92631	1.0796	2.6542	52
9	.37703	.40707	2.4566	.92620	1.0797	2.6523	51
10	.37730	.40741	2.4545	.92609	1.0798	2.6504	50
11	.37757	.40775	2.4525	.92598	1.0799	2.6485	49
12	.37784	.40809	2.4504	.92587	1.0801	2.6466	48
13	.37811	.40843	2.4484	.92576	1.0802	2.6447	47
14	.37838	.40877	2.4464	.92565	1.0803	2.6429	46
15	.37865	.40911	2.4443	.92554	1.0804	2.6410	45
16	.37892	.40945	2.4423	.92543	1.0806	2.6391	44
17	.37919	.40979	2.4403	.92532	1.0807	2.6372	43
18	.37946	.41013	2.4383	.92521	1.0808	2.6354	42
19	.37973	.41047	2.4362	.92510	1.0810	2.6335	41
20	.37999	.41081	2.4342	.92499	1.0811	2.6316	40
21	.38026	.41115	2.4322	.92488	1.0812	2.6298	39
22	.38053	.41149	2.4302	.92477	1.0814	2.6279	38
23	.38080	.41183	2.4282	.92466	1.0815	2.6260	37
24	.38107	.41217	2.4262	.92455	1.0816	2.6242	36
25	.38134	.41251	2.4242	.92444	1.0817	2.6223	35
26	.38161	.41285	2.4222	.92432	1.0819	2.6205	34
27	.38188	.41319	2.4202	.92421	1.0820	2.6186	33
28	.38215	.41353	2.4182	.92410	1.0821	2.6168	32
29	.38241	.41387	2.4162	.92399	1.0823	2.6150	31
30	.38268	.41421	2.4142	.92388	1.0824	2.6131	30
31	.38295	.41455	2.4122	.92377	1.0825	2.6113	29
32	.38322	.41490	2.4102	.92366	1.0827	2.6095	28
33	.38349	.41524	2.4083	.92355	1.0828	2.6076	27
34	.38376	.41558	2.4063	.92343	1.0829	2.6058	26
35	.38403	.41592	2.4043	.92332	1.0830	2.6040	25
36	.38430	.41626	2.4023	.92321	1.0832	2.6022	24
37	.38456	.41660	2.4004	.92310	1.0833	2.6003	23
38	.38483	.41694	2.3984	.92299	1.0834	2.5985	22
39	.38510	.41728	2.3964	.92287	1.0836	2.5967	21
40	.38537	.41763	2.3945	.92276	1.0837	2.5949	20
41	.38564	.41797	2.3925	.92265	1.0838	2.5931	19
42	.38591	.41831	2.3906	.92254	1.0840	2.5913	18
43	.38617	.41865	2.3886	.92243	1.0841	2.5895	17
44	.38644	.41899	2.3867	.92231	1.0842	2.5877	16
45	.38671	.41933	2.3847	.92220	1.0844	2.5859	15
46	.38698	.41968	2.3828	.92209	1.0845	2.5841	14
47	.38725	.42002	2.3808	.92198	1.0846	2.5823	13
48	.38752	.42036	2.3789	.92186	1.0848	2.5805	12
49	.38778	.42070	2.3770	.92175	1.0849	2.5788	11
50	.38805	.42105	2.3750	.92164	1.0850	2.5770	10
51	.38832	.42139	2.3731	.92152	1.0852	2.5752	9
52	.38859	.42173	2.3712	.92141	1.0853	2.5734	8
53	.38886	.42207	2.3693	.92130	1.0854	2.5716	7
54	.38912	.42242	2.3673	.92119	1.0856	2.5699	6
55	.38939	.42276	2.3654	92107	1.0857	2.5681	5
56	.38966	.42310	2.3635	92096	1.0858	2.5663	4
57	.38993	.42345	2.3616	.92085	1.0860	2.5646	3
58	.39020	.42379	2.3597	.92073	1.0861	2.5628	2
59	.39046	.42413	2.3578	.92062	1.0862	2.5611	1
60	.39073	.42447	2.3559	.92050	1.0864	2.5593	0
′	Cos	Cot	Tan	Sin	Csc	Sec	′

112° (292°) (247°) 67°

23° (203°) (336°) 156°

′	Sin	Tan	Cot	Cos	Sec	Csc	′
0	.39073	.42447	2.3559	.92050	1.0864	2.5593	60
1	.39100	.42482	2.3539	.92039	1.0865	2.5576	59
2	.39127	.42516	2.3520	.92028	1.0866	2.5558	58
3	.39153	.42551	2.3501	.92016	1.0868	2.5541	57
4	.39180	.42585	2 3483	92005	1.0869	2.5523	56
5	.39207	.42619	2.3464	.91994	1.0870	2.5506	55
6	.39234	.42654	2.3445	.91982	1.0872	2.5488	54
7	.39260	.42688	2.3426	.91971	1.0873	2.5471	53
8	.39287	.42722	2.3407	.91959	1.0874	2.5454	52
9	.39314	.42757	2.3388	.91948	1.0876	2.5436	51
10	.39341	.42791	2.3369	.91936	1.0877	2.5419	50
11	.39367	.42826	2.3351	.91925	1.0878	2.5402	49
12	.39394	.42860	2.3332	.91914	1.0880	2.5384	48
13	.39421	.42894	2.3313	.91902	1.0881	2.5367	47
14	.39448	.42929	2.3294	.91891	1.0883	2.5350	46
15	.39474	.42963	2.3276	.91879	1.0884	2.5333	45
16	.39501	.42998	2.3257	.91868	1.0885	2.5316	44
17	.39528	.43032	2.3238	.91856	1.0887	2.5299	43
18	.39555	.43067	2.3220	.91845	1.0888	2.5282	42
19	.39581	.43101	2.3201	.91833	1.0889	2.5264	41
20	.39608	.43136	2.3183	.91822	1.0891	2.5247	40
21	.39635	.43170	2.3164	.91810	1.0892	2.5230	39
22	.39661	.43205	2.3146	.91799	1.0893	2.5213	38
23	.39688	.43239	2.3127	.91787	1.0895	2.5196	37
24	.39715	.43274	2.3109	.91775	1.0896	2.5180	36
25	.39741	.43308	2.3090	.91764	1.0898	2.5163	35
26	.39768	.43343	2.3072	.91752	1.0899	2.5146	34
27	.39795	.43378	2.3053	.91741	1.0900	2.5129	33
28	.39822	.43412	2.3035	.91729	1.0902	2.5112	32
29	.39848	.43447	2.3017	.91718	1.0903	2.5095	31
30	.39875	.43481	2.2998	.91706	1.0904	2.5078	30
31	.39902	.43516	2.2980	.91694	1.0906	2.5062	29
32	.39928	.43550	2.2962	.91683	1.0907	2.5045	28
33	[illegible]	.43585	2.2944	.91671	1.0909	2.5028	27
34	.39982	.43620	2.2925	.91660	1.0910	2.5012	26
35	.40008	.43654	2.2907	.91648	1.0911	2.4995	25
36	.40035	.43689	2.2889	.91636	1.0913	2.4978	24
37	.40062	.43724	2.2871	.91625	1.0914	2.4962	23
38	.40088	.43758	2.2853	.91613	1.0915	2.4945	22
39	.40115	.43793	2.2835	.91601	1.0917	2.4928	21
40	.40141	.43828	2.2817	.91590	1.0918	2.4912	20
41	.40168	.43862	2.2799	.91578	1.0920	2.4895	19
42	.40195	.43897	2.2781	.91566	1.0921	2.4879	18
43	.40221	.43932	2.2763	.91555	1.0922	2.4862	17
44	.40248	.43966	2.2745	.91543	1.0924	2.4846	16
45	.40275	.44001	2.2727	.91531	1.0925	2.4830	15
46	.40301	.44036	2.2709	.91519	1.0927	2.4813	14
47	.40328	.44071	2.2691	.91508	1.0928	2.4797	13
48	.40355	.44105	2.2673	.91496	1.0929	2.4780	12
49	.40381	.44140	2.2655	.91484	1.0931	2.4764	11
50	.40408	.44175	2.2637	.91472	1.0932	2.4748	10
51	.40434	.44210	2.2620	.91461	1.0934	2.4731	9
52	.40461	.44244	2.2602	.91449	1.0935	2.4715	8
53	.40488	.44279	2.2584	.91437	1.0936	2.4699	7
54	.40514	.44314	2.2566	.91425	1.0938	2.4683	6
55	.40541	.44349	2.2549	.91414	1.0939	2.4667	5
56	.40567	.44384	2.2531	.91402	1.0941	2.4650	4
57	.40594	.44418	2.2513	.91390	1.0942	2.4634	3
58	.40621	.44453	2.2496	.91378	1.0944	2.4618	2
59	.40647	.44488	2.2478	.91366	1.0945	2.4602	1
60	.40674	.44523	2.2460	.91355	1.0946	2.4586	0
′	Cos	Cot	Tan	Sin	Csc	Sec	′

113° (293°) (246° 66°

Note the degrees listed on the top and bottom

The Pythagorean Theorem gives us the third side

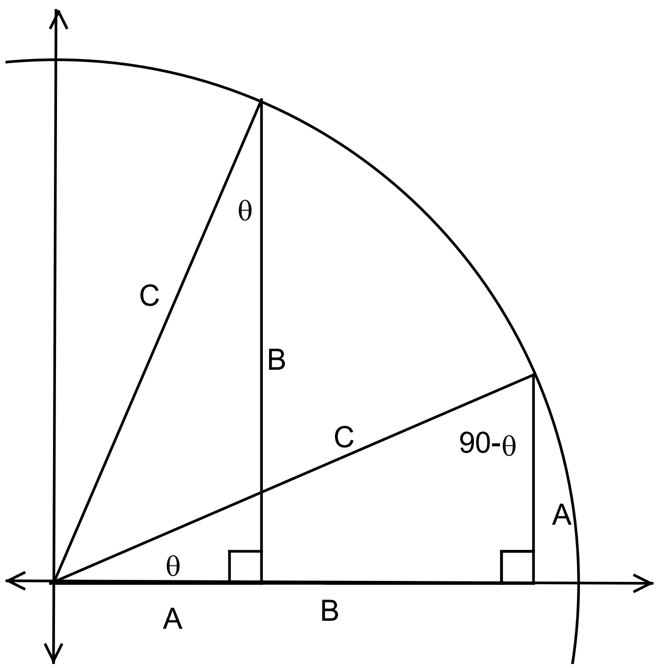
$$\sqrt{10^2 - 4^2} = \sqrt{84} \approx 9.16$$

The last angle can be found easily since it is a complementary angle

$$90^\circ - 66.42182^\circ \approx 23.57818^\circ$$

What's a complimentary angle?

Complementary Angles



Note that:

$$\sin(\theta)=\frac{A}{C} \qquad \cos(\theta)=\frac{B}{C}$$

$$\cos\left(90^{\circ}-\theta\right)=\frac{A}{C} \quad \sin\left(90^{\circ}-\theta\right)=\frac{B}{C}$$

So we have the following Identities

$$\sin(90-\theta)=\cos(\theta)$$
$$\cos(90-\theta)=\sin(\theta)$$

So we really only need to know the sines and cosines of the angles between 0° and 45°.

HW: 6.2 3,4,9, 11a, 11b, 15, 16, 21, 31, 39, 47