Trigonometry 15 Mathematics 108

Polar Coordinates

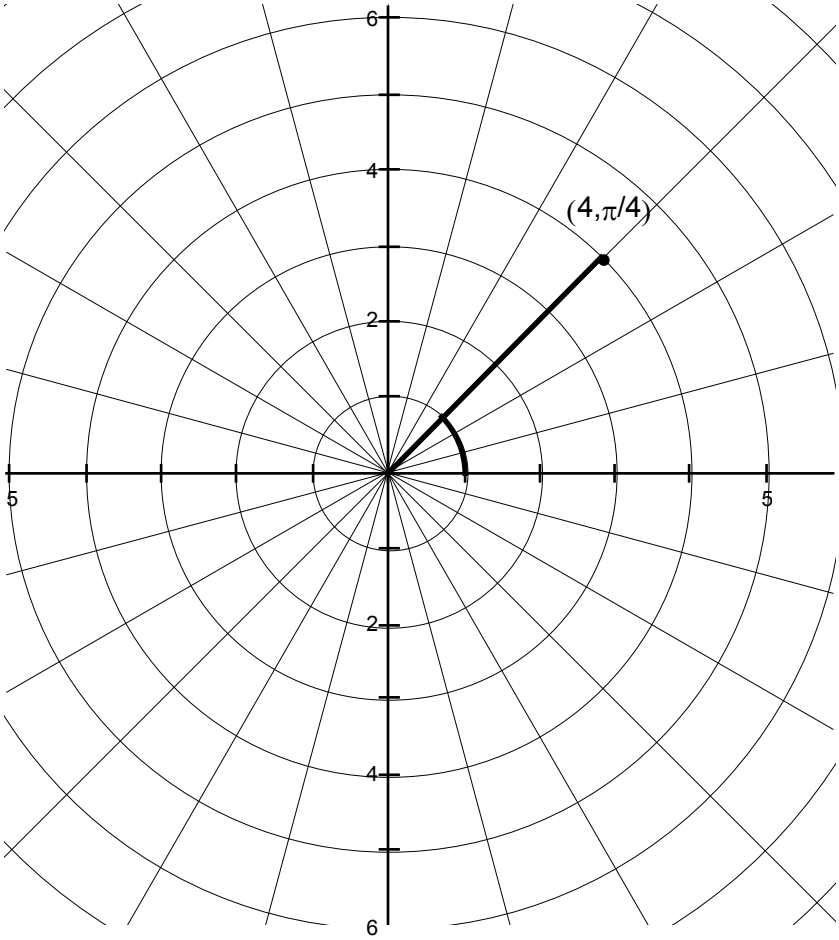
There is more than one way to setup coordinates for a point on a plane.
We are already very familiar with Cartesian coordinates, the x, y coordinates we usually work with.

An alternative is called a polar coordinate system.

Polar coordinates use two values, an angle and a radius, (R,θ)
The angle is measure from the x axis, and the radius is the distance from the origin (0,0).

Your calculator is capable of graphing using polar coordinates in the "POL" mode.

This is what the graph of a point on a polar graph looks like.

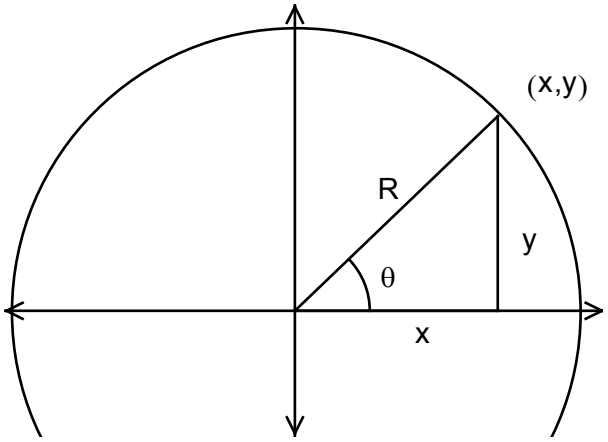


One important difference between Cartesian and Polar coordinates, is that Polar coordinates are not unique. For example:

$(3,0) \equiv (3,2\pi)$

If R=0 then the point is the origin for any value of the angle

It's important to know how to convert between Polar and Cartesian coordinates for which we using the trigonometric sine, cosine and tangent functions.



Converting Coordinates

Looking at this diagram, it should be obvious that converting from (R,θ) to (x,y) we have

$x = R\cos\theta$
$y = R\sin\theta$

Example: Convert $\left(4,\frac{2\pi}{3}\right)$ to Cartesian coordinates

$$x = 4\cos\left(\frac{2\pi}{3}\right) = 4\cdot\frac{-1}{2} = -2$$
$$y = 4\sin\left(\frac{2\pi}{3}\right) = 4\cdot\frac{\sqrt{3}}{2} = 2\sqrt{3}$$

Converting from Cartesian is not quite as straight forward.

From the Pythagorean theorem we can see that

$R^2 = x^2 + y^2$

so

$R = \sqrt{x^2 + y^2}$

We can set $\theta = \arctan\left(\frac{y}{x}\right)$ however this creates an ambiguity.

If for example *x*=*y*, is the angle $\frac{\pi}{4}$ or $\frac{5\pi}{4}$?

The quadrant that θ is in can be determined by the signs of *x* and *y*.

As a note to those of you who are computer science majors, most languages have a function

arctan2(*x*,*y*) that returns an angle with the correct sign.

Example:

Convert $(2,-2)$ to Polar form

$R = \sqrt{4+4} = 2\sqrt{2}$

$\theta = \tan^{-1}(-1) = \frac{3\pi}{4}$

We see that $(2,-2)$ is in the fourth quadrant so the coordinates are

$\left(2\sqrt{2}, \frac{7\pi}{4}\right)$

HW: 5-9 (On one graph), 17-20, 29-32, 37-40