Section 10-4 Matrices

A matrix is a set of numbers divided into rows and columns of a specified size n x m.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ or } \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The numbers are known as entries.

Matrices are equal when they have the same number of rows and columns and each entry is equal.

$$\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix} \neq \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} \neq \begin{pmatrix} 5 & 2 & 0 \\ 3 & 1 & 0 \end{pmatrix}$$

Matrix operations

Two matrices can be added if they a have the same number of rows and columns by adding each respective entry.

$$\begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 5 & 5 \end{pmatrix}$$

Matrices can be multiplied by a constant.

$$4\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 8 \\ 12 & 8 \end{pmatrix}$$

Note that this is similar to a vector.

Matrix addition has properties that are similar to the addition of real numbers.

1. Closure
2. Associative
3. Commutive
4. Identity
   $I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
5. Every matrix has an inverse such that
   $A + {}^{-}A = I$

Multiplication of a constant distributes.

$$C\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix}\right] = C\begin{pmatrix} a & b \\ c & d \end{pmatrix} + C\begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

Matrix Multiplication

If you multiply to matrices of sizes m x n and n x k you get a matrix of size m x k

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 21 & 28 \\ 49 & 64 \end{pmatrix}$$

If we limit ourselves to square matrixes, we find some new properties:

1. Closure
2. Associative $(A \cdot B) \cdot C = A \cdot (B \cdot C)$
3. Identity $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
4. Inverse: $A \cdot A^{-1} = I$

Like real numbers for which zero does not have an identity, some matrices do not have an inverse. These are called singular matrices.

What can we do with Matrices?

Matrices can act as transformations on vectors by multiplication:

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x+y \\ 2y+x \end{pmatrix}$$

This is a translation of the vector.

If we take three vectors as the vertices of a triangle and we translate it this way we get a translated triangle.

Another type of translation is a rotation. This is what a rotation matrix in 2D looks like:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Similarly, we can create a matrix that rotations 3D vectors on three axis.

Computers, 3D graphics and Video Games

Doom:

https://www.youtube.com/watch?v=Q4GiCg_m8wA

3D video games have to translate a 3D world or model to 2D on a screen. In order to do this, today they have special hardware known as a GPU, Graphics Processing Unit.

Every pixel on the screen must be calculated and put on the screen 30-240 times a second. A programmer has to program in the model to the computer but instead of moving things around, only a

4 x 4 matrix needs to be sent to the GPU.

Here is an example of something simpler that I programmed.

https://www.youtube.com/watch?v=Ny_56ybX7-k

Talk about program and Lasik Surgery.