

Lesson Plan 9 Inverse Trig Functions Continued Math 48C Mitchell Schoenbrun

- 1) Attendance
- 2) Hand out homework & Quiz
- 3) Questions about homework and Quiz?

Trigonometric Functions

Function	Domain	Range
Sine	\mathbb{R}	$[-1,1]$
Cosine	\mathbb{R}	$[-1,1]$
Tangent	$x \in \mathbb{R} : \cos(x) \neq 0$	$(-\infty, +\infty)$
Cotangent	$x \in \mathbb{R} : \sin(x) \neq 0$	$(-\infty, +\infty)$
Secant	$x \in \mathbb{R} : \cos(x) \neq 0$	$(-\infty, -1] \cup [1, +\infty)$
Cosecant	$x \in \mathbb{R} : \sin(x) \neq 0$	$(-\infty, -1] \cup [1, +\infty)$

Inverse Trigonometric functions

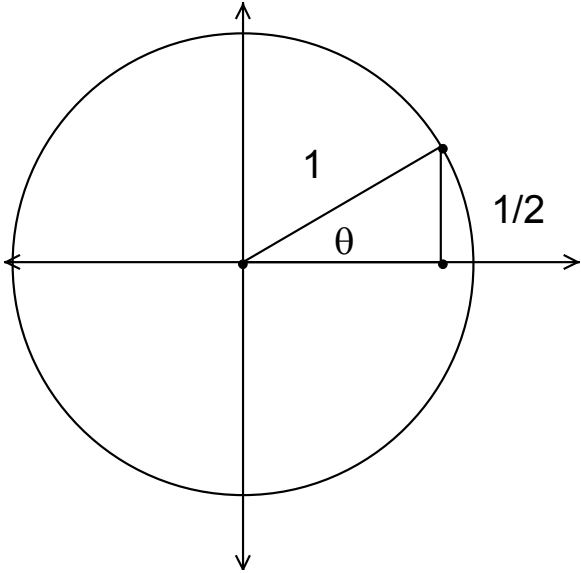
Function	Domain	Range
Inverse Sine	$[-1,1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Inverse Cosine	$[-1,1]$	$[0, \pi]$
Inverse Tangent	$(-\infty, +\infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Inverse Cotangent	$(-\infty, +\infty)$	$[0, \pi]$
Inverse Secant	$(-\infty, -1] \cup [1, +\infty)$	$[0, \pi]$
Inverse Cosecant	$(-\infty, -1] \cup [1, +\infty)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Note the Range of a function becomes the Domain of it's inverse function.
 The Range of an inverse function is smaller than the original function.

1) Solving Simple Inverse Trig problems

Exact!

Example: What is the angle whose sine is $\frac{1}{2}$, $\sin^{-1}\left(\frac{1}{2}\right) = \underline{\hspace{10em}}$



We can see immediately that this is a 30/60/90 triangle, so the angle must be 30° .

This is the principle value of $\sin^{-1}\left(\frac{1}{2}\right)$

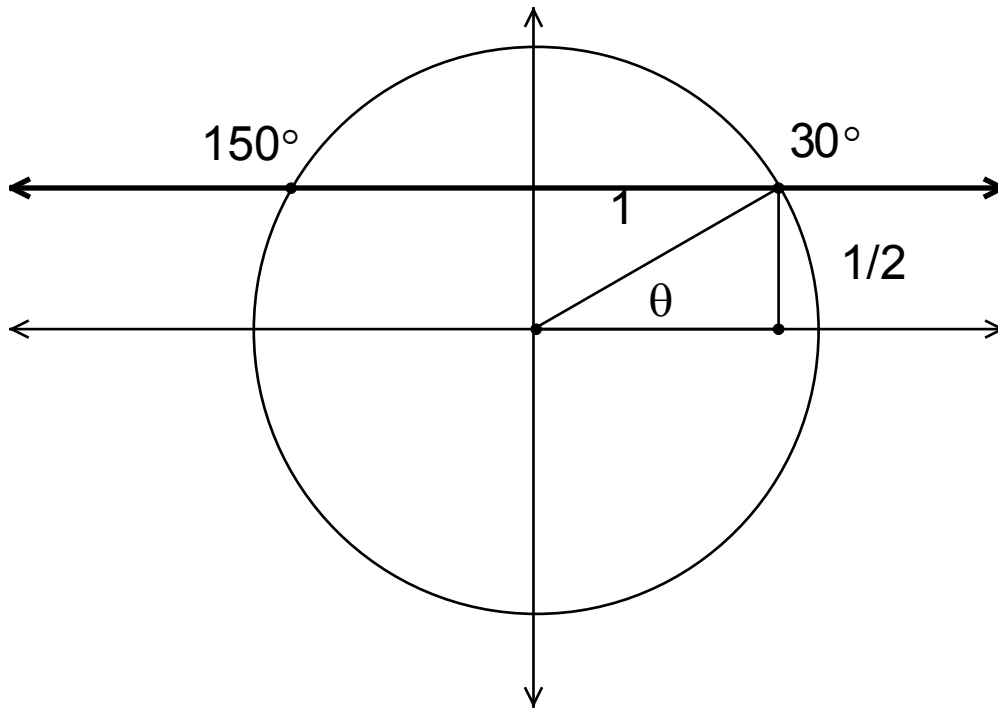
The sine is also positive in quadrant II so we also have $\sin^{-1}\left(\frac{1}{2}\right) = 180^\circ - 30^\circ = 150^\circ$.

Finally, we can find all possible solutions by adding $2\pi n$ to each of these.

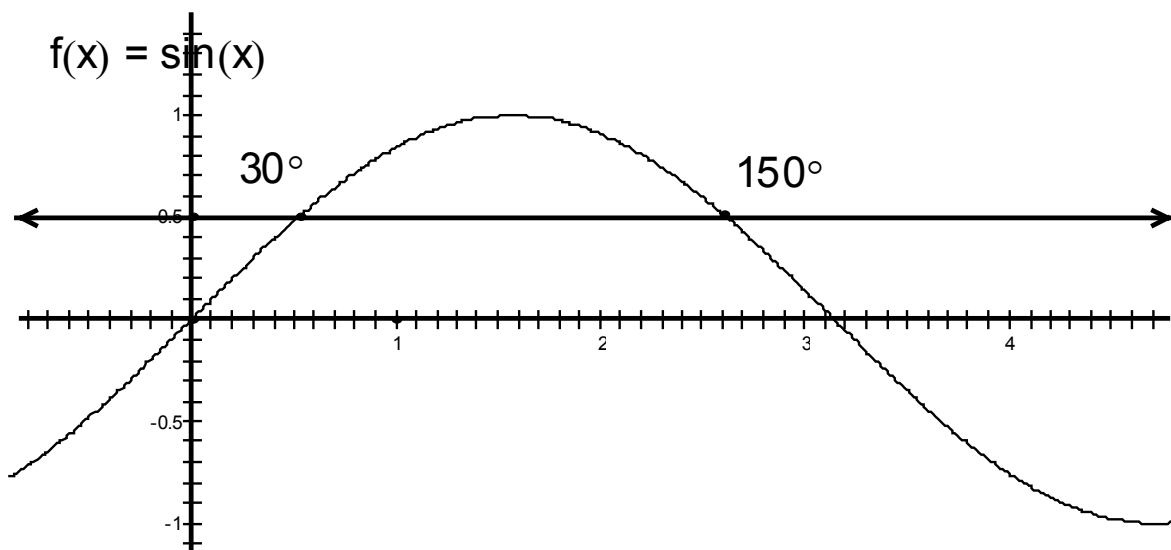
$$\sin^{-1}\left(\frac{1}{2}\right) = \{30^\circ + 360^\circ n, 150^\circ + 360^\circ n : n \in \mathbb{Z}\}$$

Two Graphical ways to interpret this last result:

Sine is the Y coordinate in the unit circle view so we see where a Horizontal line intersects the circle



Here the output from sine is on the Y axis so again a horizontal line intersects as the solution



Inexact, using a calculator

Example: What is the angle whose sine is .356?, $\sin^{-1}(.356) = \underline{\hspace{2cm}}$

Set your Calculator's mode to Degrees.

FOR INVERSE FUNCTIONS THE MODE DETERMINES THE OUTPUT~

$$\sin^{-1}(.356) = 20.85^\circ,$$

As in the previous example $180^\circ - 20.85^\circ = 159.15^\circ$.

$$\sin^{-1}(.356) = \{20.85^\circ + 360^\circ n, 159.15^\circ + 360^\circ n : n \in \mathbb{Z}\}$$

TRY FIRST 5 Problem on hand out

Go Over these problems

Finding the Inverse for $\text{ctn}()$, $\text{sec}()$ and $\text{csc}()$ with a Calculator!

Note that your calculator does not have a button for Inverse Cotangent, Secant or Secant.

Example: Find $\text{ctn}^{-1}(-3.8) = ?$

$$\text{ctn}^{-1}(x) = y \quad (\text{Take the Inverse of both sides})$$

$$x = \text{ctn}(y) \quad (\text{Take the reciprocal of both sides})$$

$$\frac{1}{x} = \frac{1}{\text{ctn}(y)} = \tan(y) \quad (\text{Find the inverse tangent of both sides})$$

$$\tan^{-1}\left(\frac{1}{x}\right) = y \quad (\text{Now substitute } y \text{ from the first equation})$$

$$\text{ctn}^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right) \quad (\text{So we can find the inverse cotangent of } x \text{ by finding the inverse tangent of } 1/x)$$

Similarly

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right) \quad \text{and} \quad \csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$$

TRY Problems 6 and 7 on the hand out

Go Over these problems

More complicated Problems:

Find all solutions to the following: $\sin(3\theta) = 0.469$ for $0^\circ \leq \theta \leq 360^\circ$.

First note that $\sin^{-1}(0.469) = 27.97^\circ$ In Quadrant I

$$\text{So } \theta = \frac{27.97^\circ}{3} = 9.32$$

But the sine function is also positive in Quadrant II giving the solution:

$$\frac{180^\circ - 27.97^\circ}{3} = 50.68$$

Next we can try

$$\frac{360^\circ + 27.97^\circ}{3} = 129.32$$

And

$$\frac{360^\circ + 180^\circ - 27.97^\circ}{3} = 170.68$$

Continuing

$$\frac{2(360^\circ) + 27.97^\circ}{3} = 249.32$$

$$\frac{2(360^\circ) - 27.97^\circ}{3} = 290.68$$

$$\frac{3(360^\circ) + 27.97^\circ}{3} = 369.3 > 360 \text{ So we are done:}$$

$$\theta = \{9.32, 50.68, 129.32, 170.68, 249.32, 290.68\}$$

You might notice that 9.32, 129.32, and 249.32 differ by $360/3 = 120$ and 50.68, 170.68 and 290.68 also differ by 120.

Is this a coincidence?

Do problem 8 on the Handout
Go Over the problems

Alternative Notation:

$\sin^{-1} \theta$	$\arcsin \theta$
$\cos^{-1} \theta$	$\arccos \theta$
$\tan^{-1} \theta$	$\arctan \theta$

Note that some texts will make the following distinction

$\arcsin \theta$ is a function (one output) with a range $[-90^\circ, 90^\circ]$

$\text{Arcsin } \theta$ is a relation with an infinite number of outputs.