Lesson Plan 18 Trigonometric Identities III, Math 48C Mitchell Schoenbrun

1) Attendance

VERIFYING IDENTIES

Example:

Verify $\dfrac{\sin\theta}{1-\cos\theta} = \dfrac{1+\cos\theta}{\sin\theta}$

Try cross multiplying:

$\sin^2\theta = 1-\cos^2\theta = \sin^2\theta$

BUT THIS ASSUME THE EQUALITY, WHICH IS NOT A VALID WAY TO VERIFY THE IDENTITY

Instead:

$$\left| \begin{array}{ccc} \textit{Expression A} & = & \textit{Expression B} \\ \vdots & & \vdots \\ \textit{Expression X} & = & \textit{Expression X} \end{array} \right.$$

So multiply Left side by $\dfrac{1+\cos\theta}{1+\cos\theta}$

$$\frac{\sin\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta} = \frac{1+\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta} = \frac{1+\cos\theta}{\sin\theta}$$

$$\frac{1+\cos\theta}{\sin\theta} = \frac{1+\cos\theta}{\sin\theta}$$

Show the following Examples:

$$\csc\theta\tan\theta = \cos\theta\sec^2\theta$$

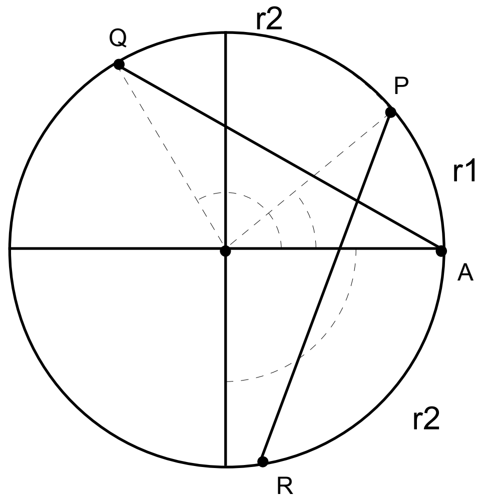
$$\frac{\sec(-\theta)}{\tan(-\theta)} = -\csc\theta$$

$$\cot^2\theta = \csc^2\theta - \cot\theta\tan\theta$$

Have students do the first 3 problems on the handout

Go Over problems

Break



Note that the way this is constructed $\widehat{AQ} = \widehat{PR}$ and so $\overline{AQ} = \overline{PR}$

The coordinates of A are (1, 0)
The coordinates of Q are (cos(r1+r2), sin(r1+r2))
The coordinates of P are (cos(r1), sin(r1))
The coordinates of R are (cos(-r2), sin(-r2))

Using the distance formula and setting $\overline{AQ} = \overline{PR}$ we have:

$$\sqrt{\left[\cos\left(r_1+r_2\right)-1\right]^2+\left[\sin\left(r_1+r_2\right)-0\right]^2} = \sqrt{\left[\cos\left(r_1\right)-\cos\left(-r_2\right)\right]^2+\left[\sin\left(r_1\right)-\sin\left(-r_2\right)\right]^2}$$

Squaring both sides:

$$\left[\cos\left(r_1+r_2\right)-1\right]^2+\sin^2\left(r_1+r_2\right)=\left[\cos\left(r_1\right)-\cos\left(r_2\right)\right]^2+\left[\sin\left(r_1\right)+\sin\left(r_2\right)\right]^2$$

Expanding:

$$\cos^2\left(r_1+r_2\right)-2\cos\left(r_1+r_2\right)+1+\sin^2\left(r_1+r_2\right)=$$
$$\cos^2\left(r_1\right)-\cos\left(r_1\right)\cos\left(r_2\right)+\cos^2\left(r_2\right)+\sin^2\left(r_1\right)+2\sin\left(r_1\right)\sin\left(r_2\right)+\sin^2\left(r_2\right)$$

Simplifying:

$$2-2\cos\left(r_1+r_2\right)=2-\cos\left(r_1\right)\cos\left(r_2\right)+2\sin\left(r_1\right)\sin\left(r_2\right)$$

Subtracting 2 and dividing by -2 gives the final result

$$\cos\left(r_1+r_2\right)=\cos\left(r_1\right)\cos\left(r_2\right)-\sin\left(r_1\right)\sin\left(r_2\right)$$

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

Plugging in -y for y and simplifying using odd/even identities

$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

The derivation for sin(x+y) is similar but won't be covered:

$$\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$$

and similarly

$$\sin(x-y) = \sin(x)\cos(y) - \sin(y)\cos(x)$$

To find tan(x+y):

$$\tan(x+y) = \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y}$$

$$\frac{\sin x \cos y + \sin y \cos x}{\cos x \cos y - \sin x \sin y} \cdot \frac{\frac{1}{\cos x \cos y}}{\frac{1}{\cos x \cos y}} = \frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{1 - \frac{\sin x \sin y}{\cos x \cos y}} = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

so: $\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$

Similarly:

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Do additional problems on the handout.

If time allows, go over, otherwise assign as homework.