Lesson Plan 3 - Unit Circle & Trig Functions Part 1

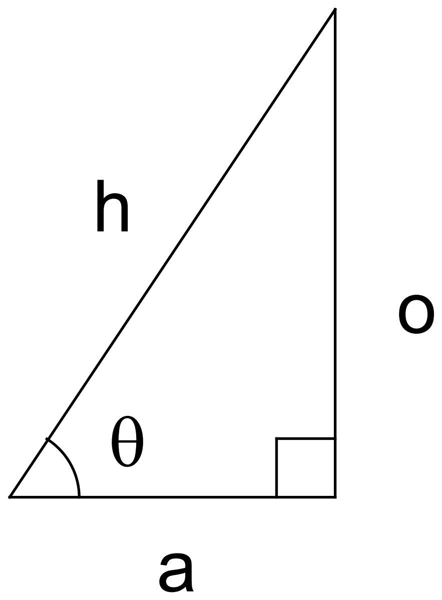
1) Take attendance

2) Homework

3) Quiz on 6.1, 8.1, 8.3 On Monday

4) Discuss Exact and Inexact answers

View of Trig functions from the point of view of a Right Triangle



h-hypotenuse

o-opposite

a-adjacent

$$\sin(\theta) = \frac{o}{h}$$

$$\cos(\theta) = \frac{a}{h}$$

$$\tan(\theta) = \frac{o}{a}$$

Memory Device: SOH-CAH-TOA (Sounds indian)

SOH (Sin = 0/H)
Sine = Opposite/Hypoteneuse

CAH (Cos=A/H)
Cosine = Adjacent/Hypoteneuse

TOA (Tan=O/A)
Tangent = Opposite/Adjacent

This is only useful when you have a right triangle. Note that $0 < \theta < \frac{\pi}{2}$.

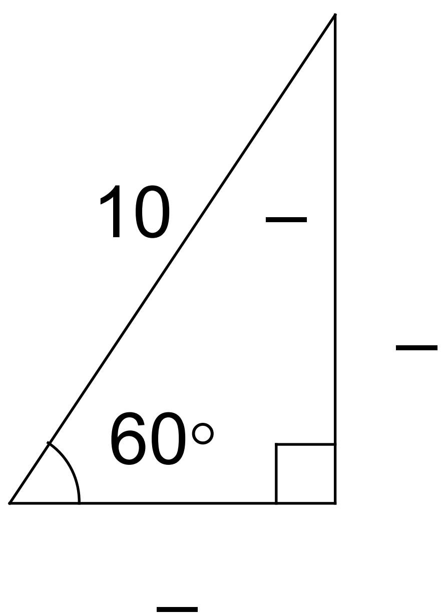
Review of Sines and Cosines

http://schoenbrun.com/foothill/math48c-2/mpeg/Ratios.mp4

What kind of problems can we solve with this?

Given any two of θ, h, a or o, we can find all missing angles and sides of the triangle.

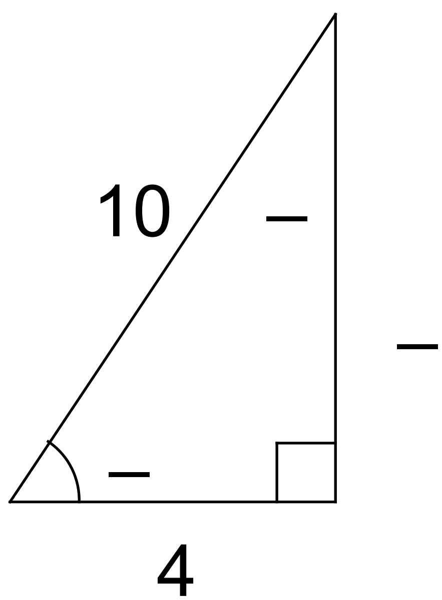
Example: Given a right triangle with hypotenuse length 10 and missing sides and $\theta = 60°$ what are the missing angles and sides?



$$\sin(60^\circ) = \sqrt{3}/2$$

$$\cos(60^\circ) = 1/2$$

Example: Given a right triangle with hypotenuse length 10 and leg 4, what are the missing angles and sides?



Finding the value of Sines and Cosines

Some ancient history

How we used to get the values of arbitrary trig functions from a table.

254 *Tables for Use in Trigonometry*

NATURAL TRIGONOMETRIC FUNCTIONS
TO FIVE PLACES

28° (208°) — (331°) **151°**

′	Sin	Tan	Cot	Cos	Sec	Csc	′
0	.46947	.53171	1.8807	.88295	1.1326	2.1301	60
1	.46973	.53208	1.8794	.88281	1.1327	2.1289	59
2	.46999	.53246	1.8781	.88267	1.1329	2.1277	58
3	.47024	.53283	1.8768	.88254	1.1331	2.1266	57
4	.47050	.53320	1.8755	.88240	1.1333	2.1254	56
5	.47076	.53358	1.8741	.88226	1.1334	2.1242	55
6	.47101	.53395	1.8728	.88213	1.1336	2.1231	54
7	.47127	.53432	1.8715	.88199	1.1338	2.1219	53
8	.47153	.53470	1.8702	.88185	1.1340	2.1208	52
9	.47178	.53507	1.8689	.88172	1.1342	2.1196	51
10	.47204	.53545	1.8676	.88158	1.1343	2.1185	50
11	.47229	.53582	1.8663	.88144	1.1345	2.1173	49
12	.47255	.53620	1.8650	.88130	1.1347	2.1162	48
13	.47281	.53657	1.8637	.88117	1.1349	2.1150	47
14	.47306	.53694	1.8624	.88103	1.1350	2.1139	46
15	.47332	.53732	1.8611	.88089	1.1352	2.1127	45
16	.47358	.53769	1.8598	.88075	1.1354	2.1116	44
17	.47383	.53807	1.8585	.88062	1.1356	2.1105	43
18	.47409	.53844	1.8572	.88048	1.1357	2.1093	42
19	.47434	.53882	1.8559	.88034	1.1359	2.1082	41
20	.47460	.53920	1.8546	.88020	1.1361	2.1070	40
21	.47486	.53957	1.8533	.88006	1.1363	2.1059	39
22	.47511	.53995	1.8520	.87993	1.1365	2.1048	38
23	.47537	.54032	1.8507	.87979	1.1366	2.1036	37
24	.47562	.54070	1.8495	.87965	1.1368	2.1025	36
25	.47588	.54107	1.8482	.87951	1.1370	2.1014	35
26	.47614	.54145	1.8469	.87937	1.1372	2.1002	34
27	.47639	.54183	1.8456	.87923	1.1374	2.0991	33
28	.47665	.54220	1.8443	.87909	1.1375	2.0980	32
29	.47690	.54258	1.8430	.87896	1.1377	2.0969	31
30	.47716	.54296	1.8418	.87882	1.1379	2.0957	30
31	.47741	.54333	1.8405	.87868	1.1381	2.0946	29
32	.47767	.54371	1.8392	.87854	1.1383	2.0935	28
33	.47793	.54409	1.8379	.87840	1.1384	2.0924	27
34	.47818	.54446	1.8367	.87826	1.1386	2.0913	26
35	.47844	.54484	1.8354	.87812	1.1388	2.0901	25
36	.47869	.54522	1.8341	.87798	1.1390	2.0890	24
37	.47895	.54560	1.8329	.87784	1.1392	2.0879	23
38	.47920	.54597	1.8316	.87770	1.1393	2.0868	22
39	.47946	.54635	1.8303	.87756	1.1395	2.0857	21
40	.47971	.54673	1.8291	.87743	1.1397	2.0846	20
41	.47997	.54711	1.8278	.87729	1.1399	2.0835	19
42	.48022	.54748	1.8265	.87715	1.1401	2.0824	18
43	.48048	.54786	1.8253	.87701	1.1402	2.0813	17
44	.48073	.54824	1.8240	.87687	1.1404	2.0802	16
45	.48099	.54862	1.8228	.87673	1.1406	2.0791	15
46	.48124	.54900	1.8215	.87659	1.1408	2.0779	14
47	.48150	.54938	1.8202	.87645	1.1410	2.0768	13
48	.48175	.54975	1.8190	.87631	1.1412	2.0757	12
49	.48201	.55013	1.8177	.87617	1.1413	2.0747	11
50	.48226	.55051	1.8165	.87603	1.1415	2.0736	10
51	.48252	.55089	1.8152	.87589	1.1417	2.0725	9
52	.48277	.55127	1.8140	.87575	1.1419	2.0714	8
53	.48303	.55165	1.8127	.87561	1.1421	2.0703	7
54	.48328	.55203	1.8115	.87546	1.1423	2.0692	6
55	.48354	.55241	1.8103	.87532	1.1424	2.0681	5
56	.48379	.55279	1.8090	.87518	1.1426	2.0670	4
57	.48405	.55317	1.8078	.87504	1.1428	2.0659	3
58	.48430	.55355	1.8065	.87490	1.1430	2.0648	2
59	.48456	.55393	1.8053	.87476	1.1432	2.0637	1
60	.48481	.55431	1.8040	.87462	1.1434	2.0627	0
′	Cos	Cot	Tan	Sin	Csc	Sec	′

118° (298°) — (241°) **61°**

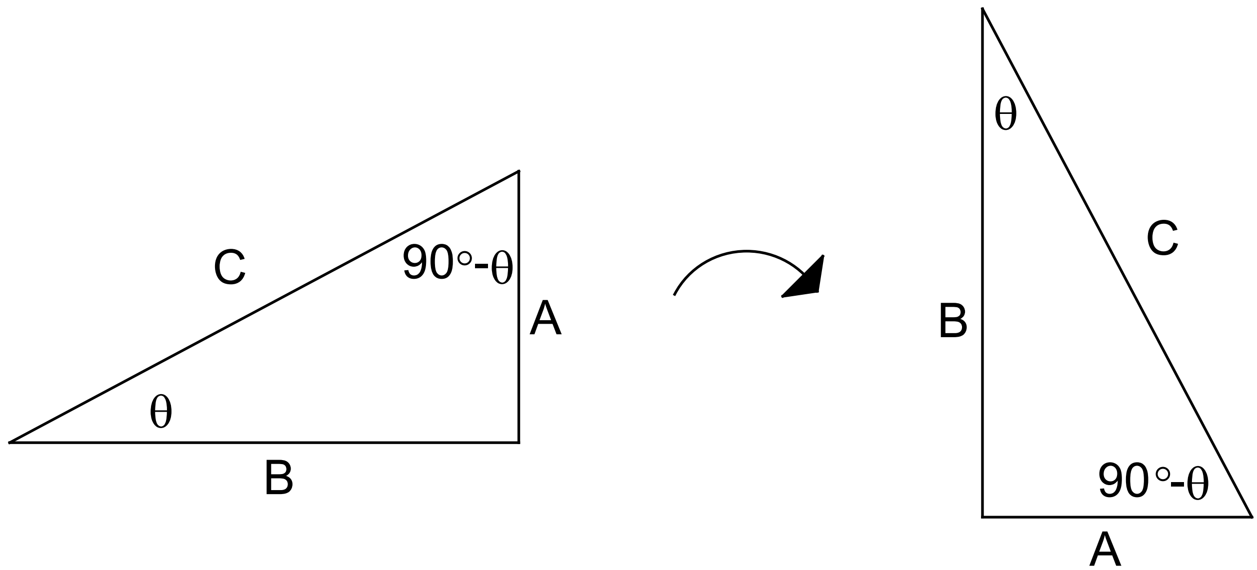
29° (209°)

′	Sin	Tan	Cot	Cos
0	.48481	.55431	1.8040	.87462
1	.48506	.55469	1.8028	.87448
2	.48532	.55507	1.8016	.87434
3	.48557	.55545	1.8003	.87420
4	.48583	.55583	1.7991	.87406
5	.48608	.55621	1.7979	.87391
6	.48634	.55659	1.7966	.87377
7	.48659	.55697	1.7954	.87363
8	.48684	.55736	1.7942	.87349
9	.48710	.55774	1.7930	.87335
10	.48735	.55812	1.7917	.87321
11	.48761	.55850	1.7905	.87306
12	.48786	.55888	1.7893	.87292
13	.48811	.55926	1.7881	.87278
14	.48837	.55964	1.7868	.87264
15	.48862	.56003	1.7856	.87250
16	.48888	.56041	1.7844	.87235
17	.48913	.56079	1.7832	.87221
18	.48938	.56117	1.7820	.87207
19	.48964	.56156	1.7808	.87193
20	.48989	.56194	1.7796	.87178
21	.49014	.56232	1.7783	.87164
22	.49040	.56270	1.7771	.87150
23	.49065	.56309	1.7759	.87136
24	.49090	.56347	1.7747	.87121
25	.49116	.56385	1.7735	.87107
26	.49141	.56424	1.7723	.87093
27	.49166	.56462	1.7711	.87079
28	.49192	.56501	1.7699	.87064
29	.49217	.56539	1.7687	.87050
30	.49242	.56577	1.7675	.87036
31	.49268	.56616	1.7663	.87021
32	.49293	.56654	1.7651	.87007
33	.49318	.56693	1.7639	.86993
34	.49344	.56731	1.7627	.86978
35	.49369	.56769	1.7615	.86964
36	.49394	.56808	1.7603	.86949
37	.49419	.56846	1.7591	.86935
38	.49445	.56885	1.7579	.86921
39	.49470	.56923	1.7567	.86906
40	.49495	.56962	1.7556	.86892
41	.49521	.57000	1.7544	.86878
42	.49546	.57039	1.7532	.86863
43	.49571	.57078	1.7520	.86849
44	.49596	.57116	1.7508	.86834
45	.49622	.57155	1.7496	.86820
46	.49647	.57193	1.7485	.86805
47	.49672	.57232	1.7473	.86791
48	.49697	.57271	1.7461	.86777
49	.49723	.57309	1.7449	.86762
50	.49748	.57348	1.7437	.86748
51	.49773	.57386	1.7426	.86733
52	.49798	.57425	1.7414	.86719
53	.49824	.57464	1.7402	.86704
54	.49849	.57503	1.7391	.86690
55	.49874	.57541	1.7379	.86675
56	.49899	.57580	1.7367	.86661
57	.49924	.57619	1.7355	.86646
58	.49950	.57657	1.7344	.86632
59	.49975	.57696	1.7332	.86617
60	.50000	.57735	1.7321	.86603
′	Cos	Cot	Tan	Sin

119° (299°)

Note the degrees listed on the top and bottom

Complementary Angles



Note that:

$$\sin(\theta) = \frac{A}{C} \qquad \cos(\theta) = \frac{B}{C}$$

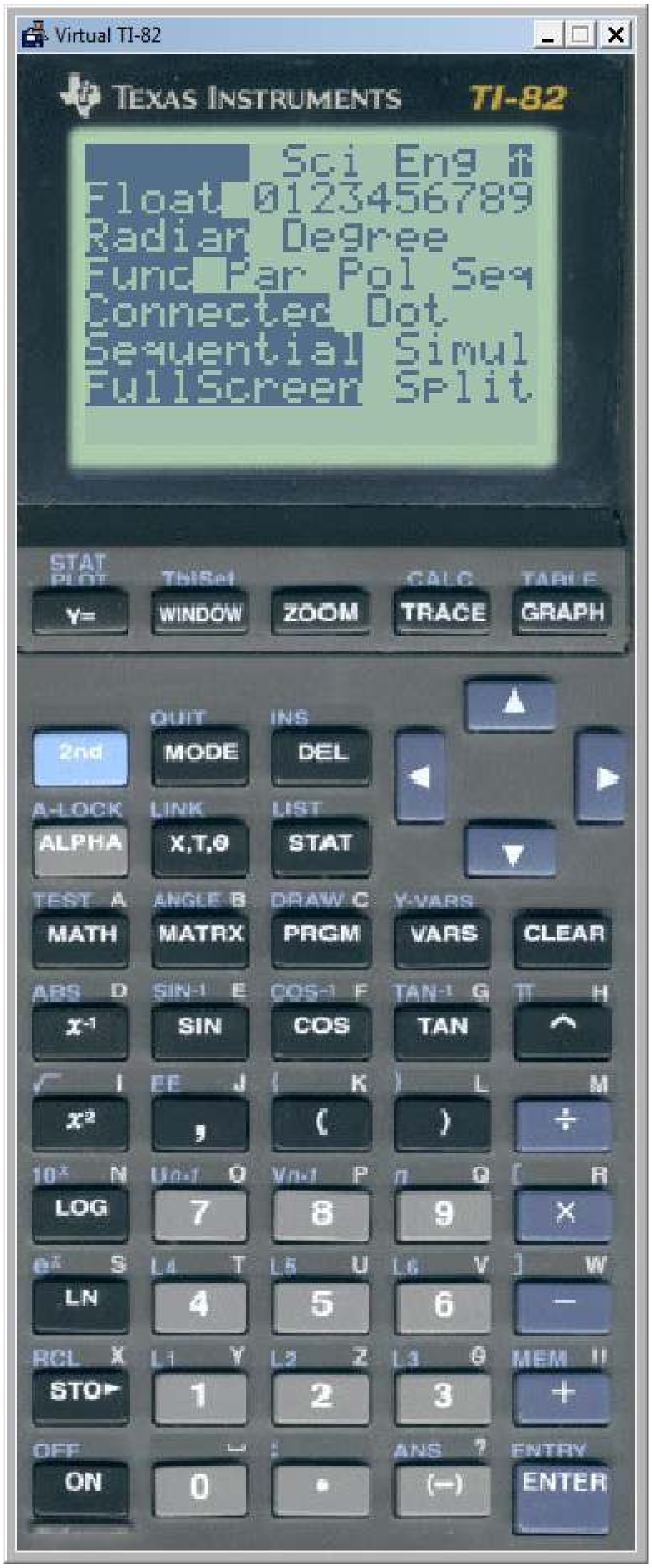
$$\cos(90^\circ - \theta) = \frac{A}{C} \quad \sin(90^\circ - \theta) = \frac{B}{C}$$

So we have the following Identities

$$\sin(90 - \theta) = \cos(\theta)$$
$$\cos(90 - \theta) = \sin(\theta)$$

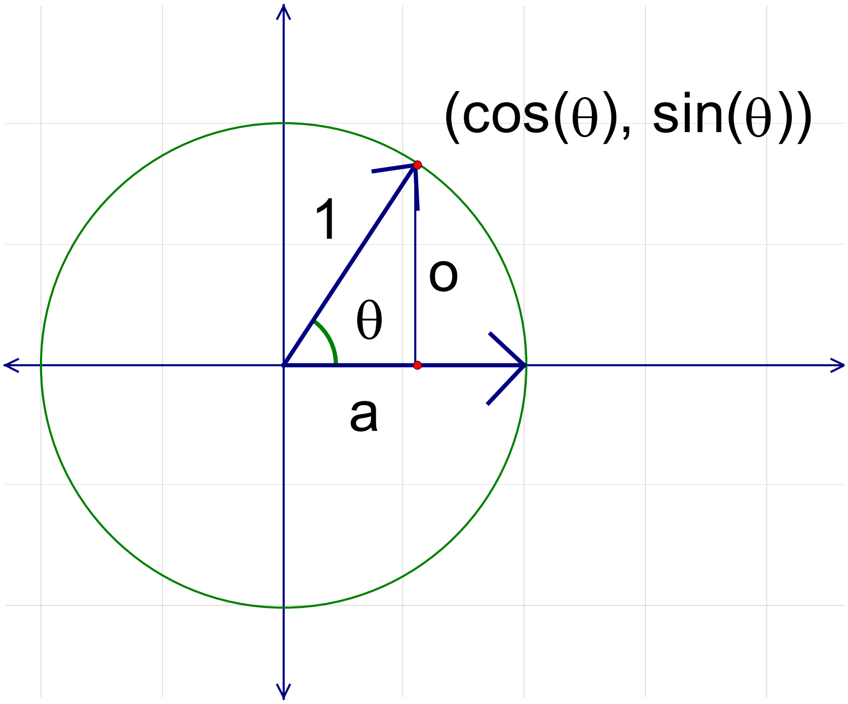
So we really only need to know the sines and cosines of the angles between 0° and 45°.

Using a Calculator to find Sines and Cosines
ALWAYS CHECK THE MODE!!!!!!!



Unit Circle View of Trig functions

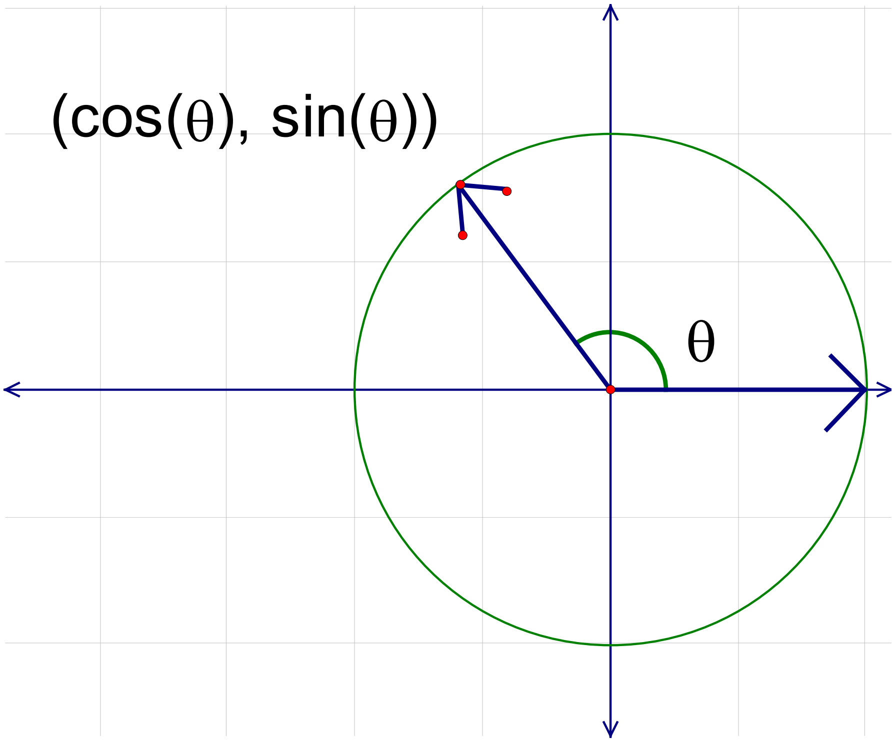
Here we redefine the sin and cosine functions as coordinates on the unit circle.



Take a look at this animation and notice that the sine and cosine function are doing the same thing, only out of sync by 90°

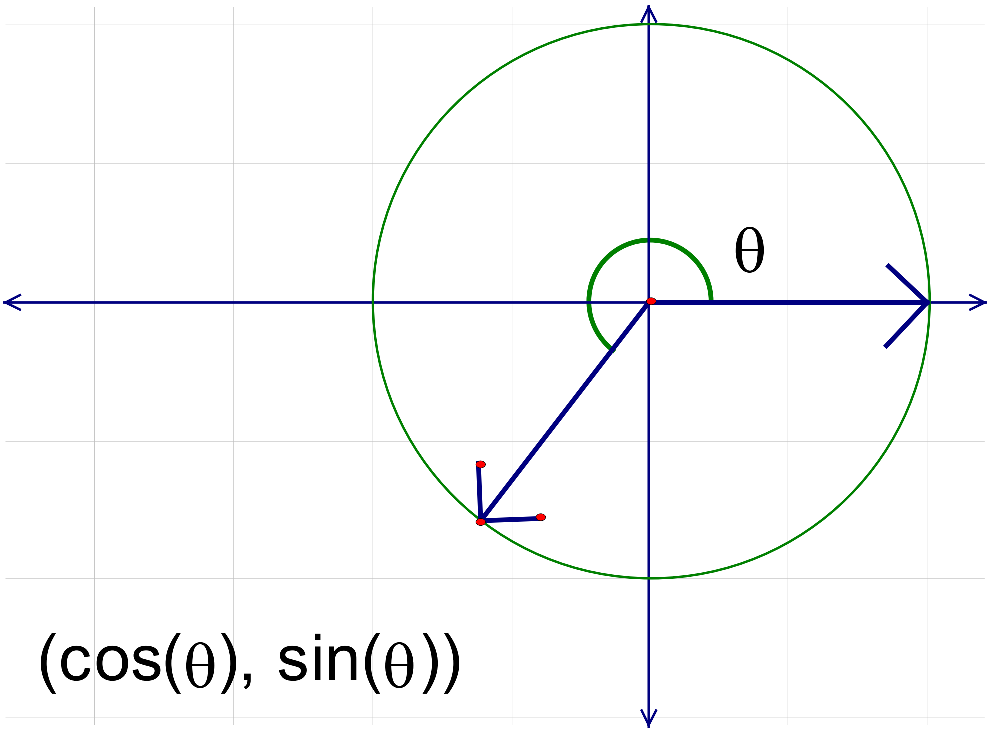
http://schoenbrun.com/foothill/math48c-2/gsps/CircularMotion.gsp

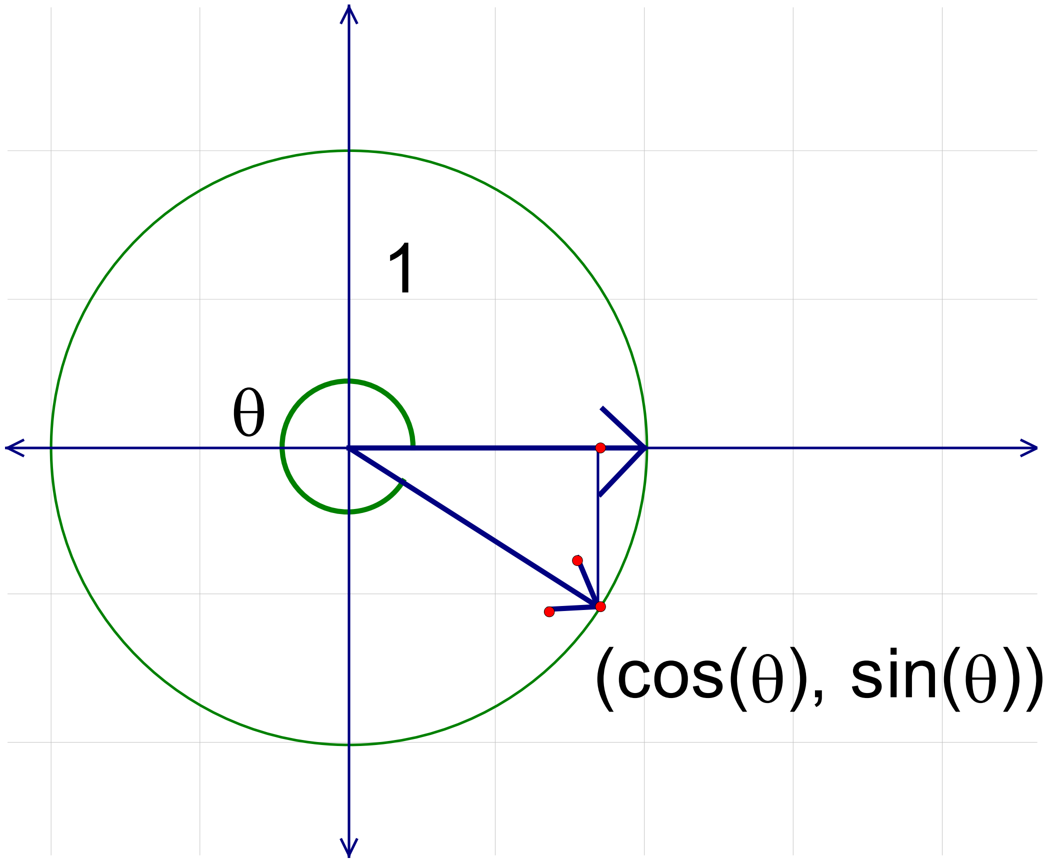
This produces the same function values as the triangle, but is not limited to $0 < \theta < \frac{\pi}{2}$



Note that in the 2nd quadrant the sine is still positive but the cosine is negative.

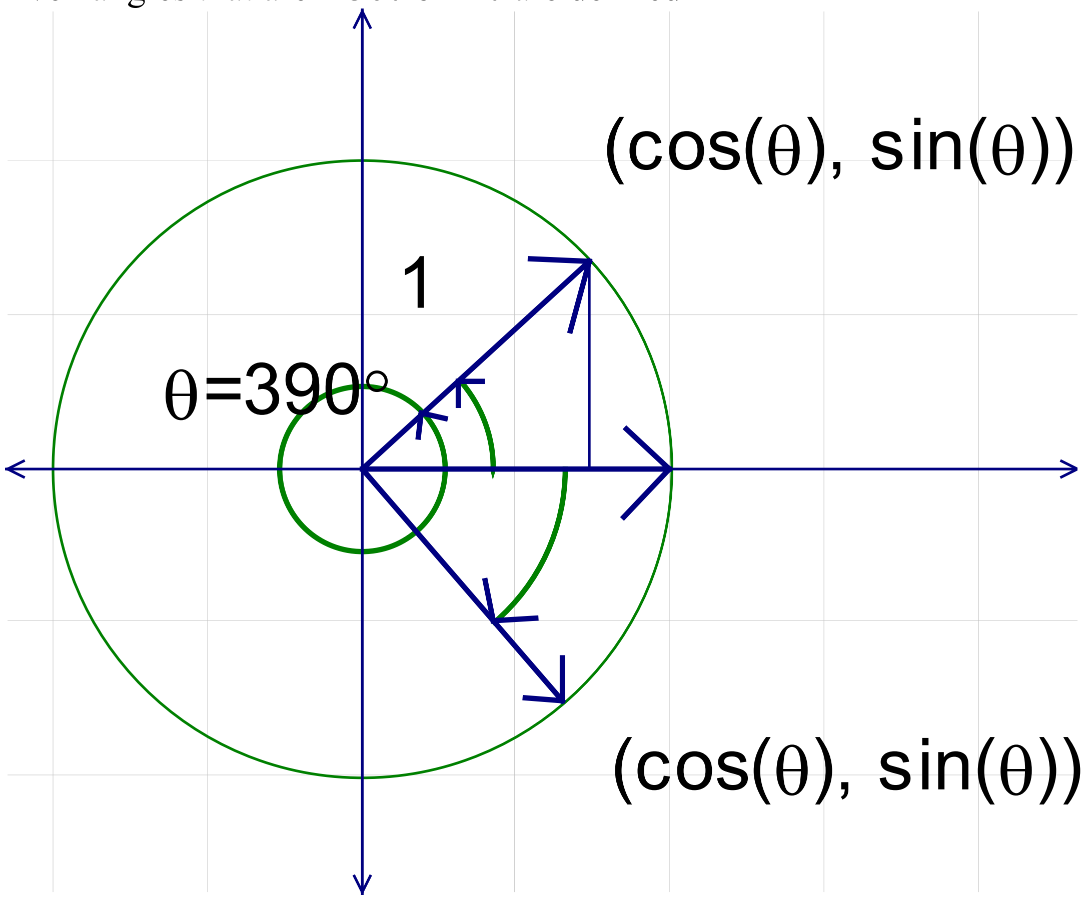
In the third quadrant both are negative



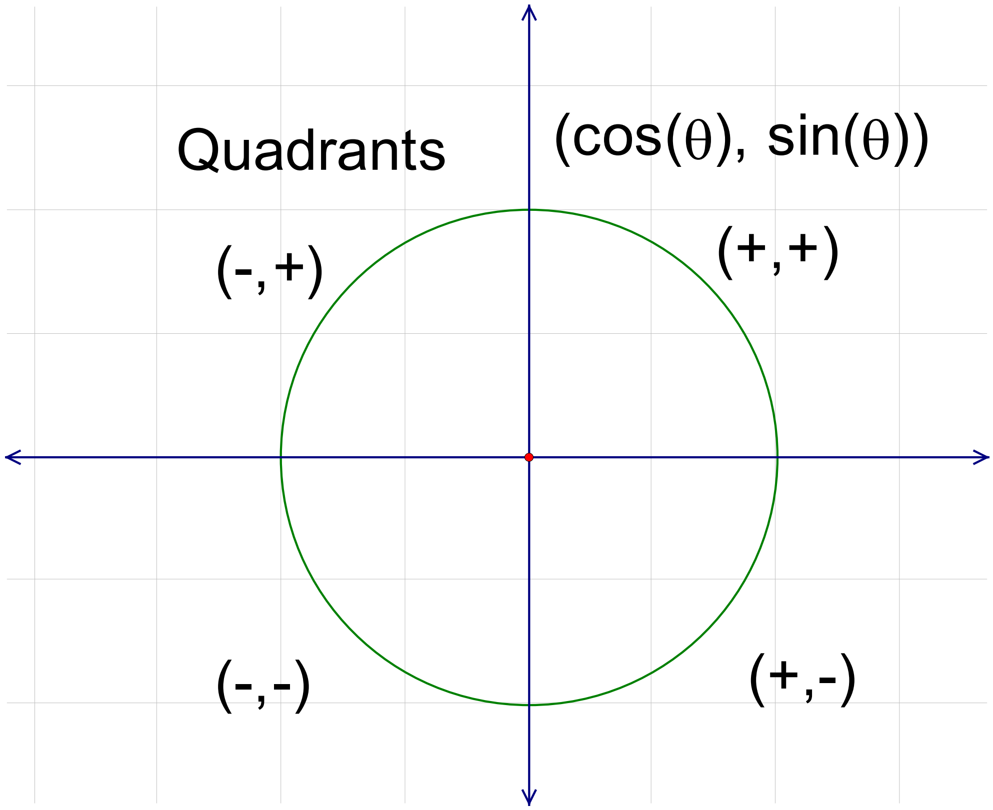


And in the 4th quadrant only the sine is negative.

Even angles that are > 360 or < 0 are defined



Here we have a map of showing what sign's of the two functions in each quadrant.



There are some important angles that now have sines and cosines:

$$\sin\left(0^\circ\right) = 0$$

$$\cos\left(0^\circ\right) = 1$$

$$\sin\left(90^\circ\right) = 1$$

$$\cos\left(90^\circ\right) = 0$$

$$\sin\left(180^\circ\right) = 0$$

$$\cos\left(180^\circ\right) = -1$$

$$\sin\left(270^\circ\right) = -1$$

$$\cos\left(270^\circ\right) = 0$$

Reference Angles

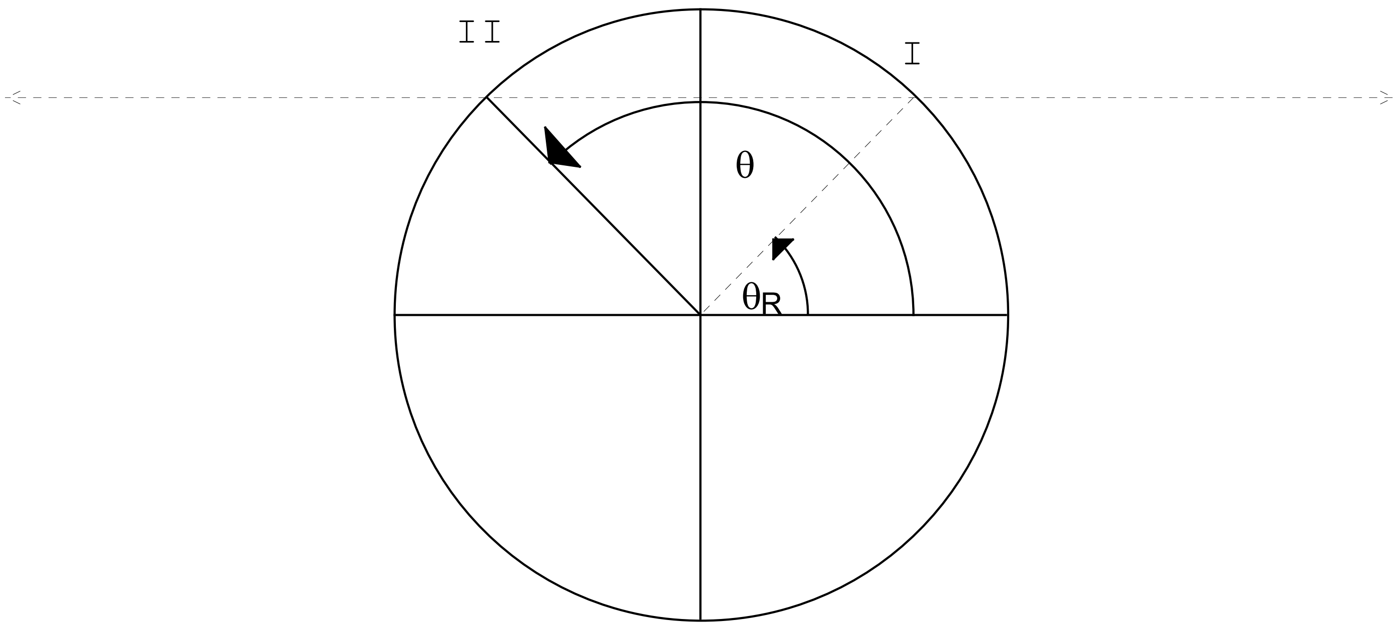
A **Reference angle** is an angle in the first quadrant.
Every angle will have a corresponding References Angle.

The trigonometric functions of an angle will have the same value as for the corresponding reference angle or it's negative.

First we look at angles $0^\circ \le \theta \le 360^\circ$

Note, that the reference angle for any angle in the first quadrant is itself.

For an angle in the 2nd quadrant, draw a line parallel to the X-axis through the point where the angle intersects the unit circle and then find where this line intersects the unit circle in the first quadrant.



Note that for the 2nd quadrant $\theta_R = 180^\circ - \theta$

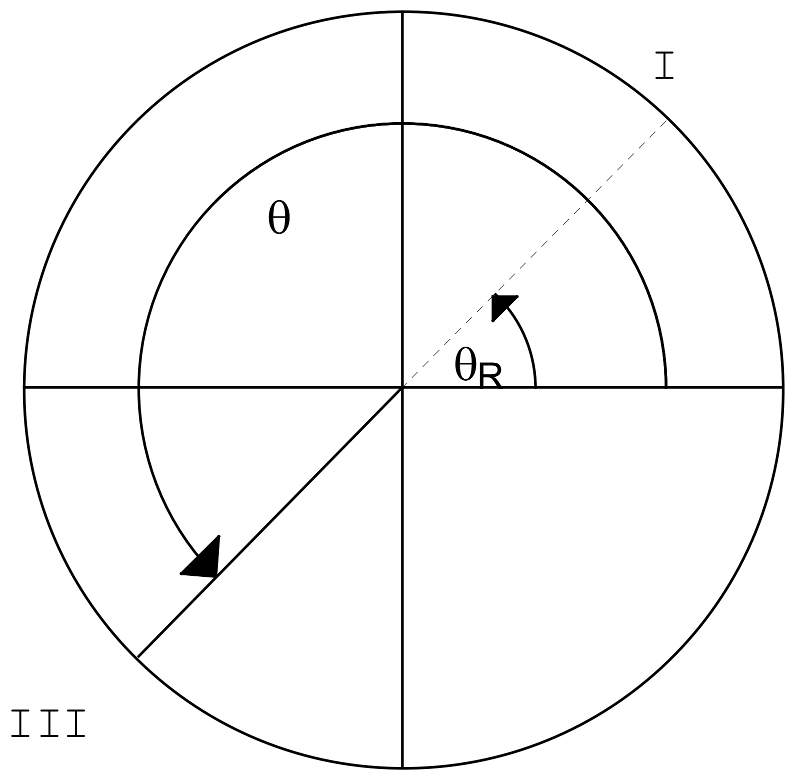
Also note that

$\sin(\theta_R) = \sin(\theta)$

but

$\cos(\theta_R) = -\cos(\theta)$

For an angle in the 3rd quadrant, extend the terminal ray in the opposite direction and find where it intersects the unit circle.



Note that for the 3rd quadrant $\theta_R = \theta - 180^\circ$

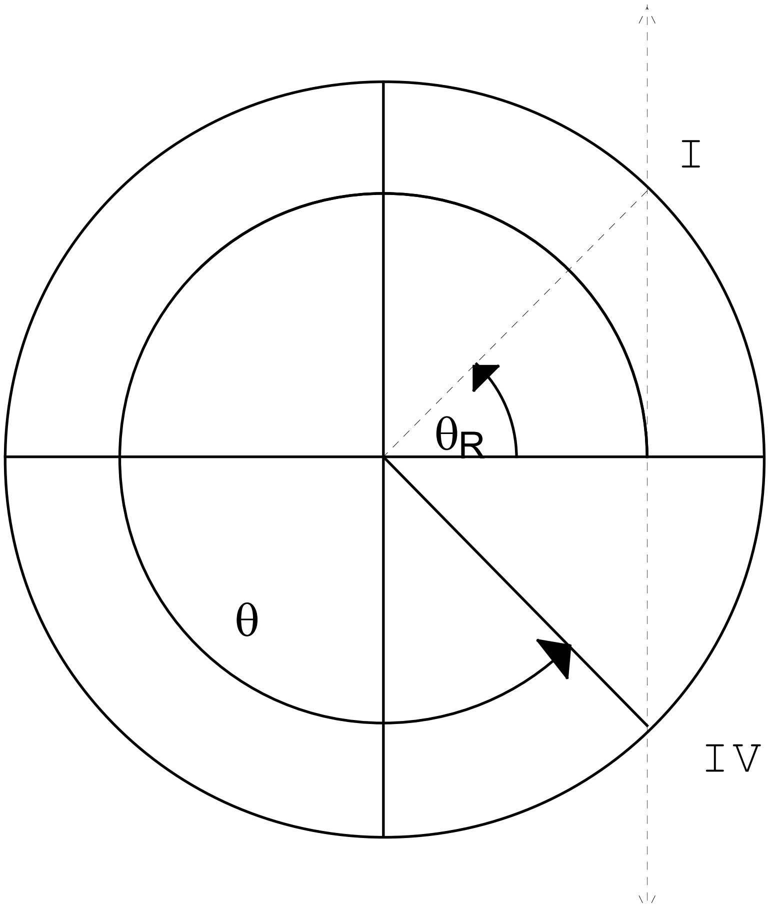
Also note that

$\sin(\theta_R) = -\sin(\theta)$

and

$\cos(\theta_R) = -\cos(\theta)$

Finally for an angle in the 4th quadrant draw a line parallel to the y axis from where the angle intersects the unit circle.



Note that for the 4th quadrant $\theta_R = 360^\circ - \theta$

Also note that

$\sin(\theta_R) = -\sin(\theta)$

but

$\cos(\theta_R) = \cos(\theta)$

For any angle $\theta < 0^\circ$ or $\theta > 360^\circ$

There is some angle θ_u, u as in unit-circle for which

$$\theta = \theta_u + n360^\circ$$

where n is an integer such that $0^\circ \leq \theta_u \leq 360^\circ$

With $\sin\left(\theta_u\right) = \sin\left(\theta\right)$

and

$$\cos\left(\theta_u\right) = \cos\left(\theta\right)$$

So for calculation purposes, we only need to know the values of the sine and cosine between 0° and 45°.

To do this first find θ_u between 0° and 360°.

Then find the Reference angle checking whether the sign changes.

Finally, if an angle is > 45°, find the other function (sine or cosine) of it's complement.

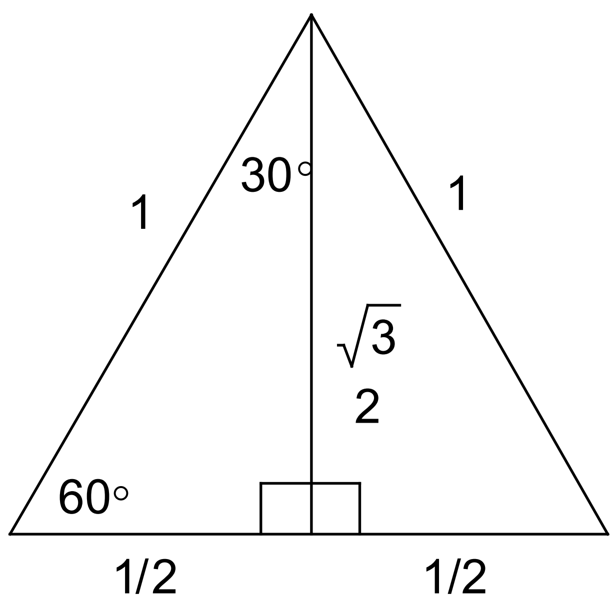
Of course if we are using a calculator, there's no reason to go through this process!

Handout Part 1!

Special Angles with Exact Values

Using our knowledge of special triangles from geometry:
30/60/90 triangles:

Take an equilateral triangle with sides 1 whose angles must all be 60°.
Drop a perpendicular from it's highest point to the base.
This divides the triangle into two congruent triangles.
By symmetry the angles of each of these triangles must be
30/60/90 degrees.
The base is 1/2 and the hypotenuse is 1 so by the Pythagorean theorem we get the second leg to be $\frac{\sqrt{3}}{2}$



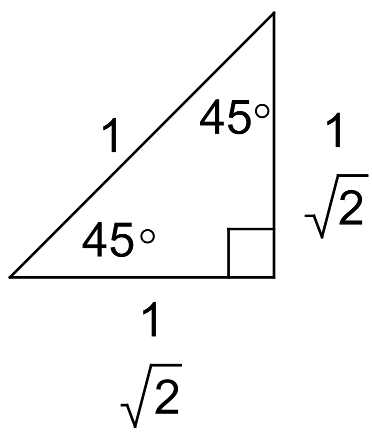
This tells us that

$$\sin\left(60^\circ\right) = \frac{\sqrt{3}}{2}$$

$$\cos\left(60^\circ\right) = \frac{1}{2}$$

Isosceles right triangles:

Given a right isosceles triangle with hypotenuse 1 we know immediately that the smaller angles are 45° and by the Pythagorean theorem, the legs are $\frac{1}{\sqrt{2}}$



This tells us that

$$\sin\left(45^\circ\right) = \frac{1}{\sqrt{2}}$$

$$\cos\left(45^\circ\right) = \frac{1}{\sqrt{2}}$$

The exact values of the sine and cosine can be determined exactly for angles that multiples of 30° and 45°.

Functions: We can find an exact expression for any multiple of 30° or 45°

Angle	Sine	Cos	Angle	Sine	Cos	Angle	Sine	Cosine	Angle	Sine	Cos
0°			90°			180°			360°		
30°			120°			210°			300°		
45°			135°			225°			315°		
60°			150°			240°			330°		

Handout Part 2!

Solutions

Angle	(cos,sin)	Angle	(cos,sin)	Angle	(cos,sin)	Angle	(cos,sin)
0°	(+1,0)	90°	(0,+1)	180°	(-1,0)	360°	(0,-1)
30°	$\left(+\frac{\sqrt{3}}{2},+\frac{1}{2}\right)$	120°	$\left(-\frac{1}{2},+\frac{\sqrt{3}}{2}\right)$	210°	$\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$	300°	$\left(+\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$
45°	$\left(+\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}\right)$	135°	$\left(-\frac{1}{\sqrt{2}},+\frac{1}{\sqrt{2}}\right)$	225°	$\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$	315°	$\left(+\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$
60°	$\left(+\frac{1}{2},+\frac{\sqrt{3}}{2}\right)$	150°	$\left(-\frac{\sqrt{3}}{2},+\frac{1}{2}\right)$	240°	$\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$	330°	$\left(+\frac{\sqrt{3}}{2},-\frac{1}{2}\right)$

Demonstrate using a calculator to get sines and cosines.

Handout Part 3

Homework:

Read Section 8.3
Problems for 1/22 on page 542 #15-18, 37, 45, 50, 51, 54