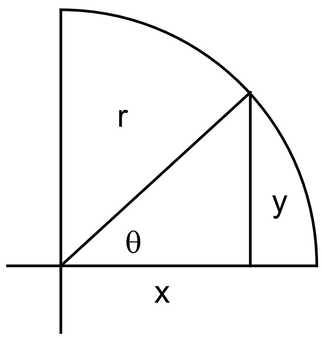
Lesson Plan 17 Trigonometric Identities II, Math 48C Mitchell Schoenbrun

1) Attendance
2) Hand back Quiz and go over
2) Go over the hand outs.

Review of identities so far

$\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)}$	$\cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$	$\sec(\theta)=\frac{1}{\cos(\theta)}$	$\csc(\theta)=\frac{1}{\sin(\theta)}$
$\sin(-\theta)=-\sin(\theta)$	$\cos(-\theta)=\cos(\theta)$	$\tan(-\theta)=-\tan(\theta)$	
$\csc(-\theta)=-\csc(\theta)$	$\sec(-\theta)=\sec(\theta)$	$ctn(-\theta)=-ctn(\theta)$	
$\sin(\theta)=\cos(90^{\circ}-\theta)$	$\cos(\theta)=\sin(90^{\circ}-\theta)$		
$\csc(\theta)=\sec(90^{\circ}-\theta)$	$\sec(\theta)=\csc(90^{\circ}-\theta)$		
$\tan(\theta)=ctn(90^{\circ}-\theta)$	$ctn(\theta)=\tan(90^{\circ}-\theta)$		

Pythagorean Identities



The Pythagorean theorem tells us that $x^2 + y^2 = r^2$

We also know that:

$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

Multiplying each of these by *r* gives

$y = r\sin\theta$ and $x = r\cos\theta$

Plugging into the first equation we get

$r^2\sin^2\theta + r^2\cos^2\theta = r^2$

Dividing through by r^2 gives us

$$\sin^2\theta + \cos^2\theta = 1$$

Question: We've shown this is true for a triangle in the first quadrant. Why is it also true in the second, third and fourth quadrant. What other value(s) of θ are we missing?

Two very useful versions of this are:

$\sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

From this we get to other useful versions:

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$ and $\cos\theta = \pm\sqrt{1-\sin^2\theta}$

Other Pythagorean Identities:

Take
$\sin^2\theta + \cos^2\theta = 1$

Divide both sides by $\cos^2\theta$ and then by $\sin^2\theta$ to get

$\tan^2\theta + 1 = \sec^2\theta$ and $ctn^2\theta + 1 = \csc^2\theta$

Example:

Solve this equation: $\cos^2\theta + \cos^2\theta\tan^2\theta = 2.6\cos\theta$

Example:

Simplify $\left[1 - \cos^2\theta\right]\left[ctn^2\theta\right]$

Example:

Simplify $\left[\sec\theta + \tan\theta\right]\left[\sec\theta - \tan\theta\right]$

VERIFYING IDENTITIES

Example:

Verify $\dfrac{\sin\theta}{1-\cos\theta}=\dfrac{1+\cos\theta}{\sin\theta}$

Try cross multiplying:

$\sin^2\theta = 1-\cos^2\theta = \sin^2\theta$

BUT THIS ASSUMES THE EQUALITY, WHICH IS NOT A VALID WAY TO VERIFY THE IDENTITY

Instead:

$$\left|\begin{array}{ccc} \textit{Expression A} & = & \textit{Expression B} \\ \vdots & & \vdots \\ \textit{Expression X} & = & \textit{Expression X} \end{array}\right.$$

So multiply Left side by $\dfrac{1+\cos\theta}{1+\cos\theta}$

$$\frac{\sin\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}=\frac{1+\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}=\frac{1+\cos\theta}{\sin\theta}$$

$$\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}$$

$$\frac{1+\cos\theta}{\sin\theta}=\frac{1+\cos\theta}{\sin\theta}$$

Show the following Examples:

$$\csc\theta\tan\theta = \cos\theta\sec^2\theta$$

$$\frac{\sec(-\theta)}{\tan(-\theta)} = -\csc\theta$$

$$\cot^2\theta = \csc^2\theta - \cot\theta\tan\theta$$

Have students do the first 3 problems on the handout

Go Over problems

Break

Handout some examples to be worked on.