Lesson Plan 16 Trigonometric Identities I, Math 48C Mitchell Schoenbrun

1) Attendance
2) Go over any homework questions
3) Quiz

Basic Identities

What is an identity:

Examples:

$3 + 1$, 2^2, $6-2$, $48/12$ are all expressions that are equivalent to 4

Likewise

$2(x-3)$ $x + x - 6$ $(4x - 12)/ 2$ are equivalent expressions.

The are always true regardless of the value of x.

They are said to form a mathematical IDENTITY.

To show that one of these is equivalent to the other we express the equivalence as a theorem and then proceed with a proof, eg.

3 + 1

2^2

6-2

48/12

$2(x-3)$

$x + x - 6$

$(4x - 12)/ 2$

To show the first two are equivalent we state a theorem:

A = 2(x-3) if and only if A = x + x - 6

We start with 2(x-3) and manipulate it until we end up with x + x - 6 as follows:

2(x-3) = 2x - 2(3) = x + x - 6, To prove the only if part we would have to start with

x + x - 6 and show 2(x-3) but in this example, we can just state that the steps are reversible.

Theorem: $A = 2(x-3)$ if and only if $A = x + x - 6$

Proof $A = 2(x-3)$ implies
$A = x + x - 6$

$2(x-3) = 2x - 2(3)$
distributive property of addition

$2x - 2(3) = x + x - 6$
distributive property again

Proof $A = x + x - 6$
implies $A = 2(x-3)$

THE STEPS ARE REVERSABLE!

When we are trying to determine if the two expressions are equal, we might start at both ends and see if we end up in the same place:

$$2(x - 3) \quad \longleftrightarrow \quad x + x - 6$$

$$2x - 6 \quad \longleftrightarrow \quad 2x - 6$$

However be careful that the steps are reversible. Otherwise you might end up with a fallacy in the proof.

A proof cannot start by assuming the result!

The book refers to following as "Fundamental Identities"

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} \qquad \csc(\theta) = \frac{1}{\sin(\theta)}$$

This is incorrect. These are Definitions, however they work just like identities.
Both expressions are always equivalent, however by definition.
A definition does not need to be proved, nor can it be.

Example 1:

Show that $\tan(\theta)\csc(\theta)\cos(\theta)=1$

Note that this "bad example" of an identity because is not true for all theta.
Can anyone tell me why? We need to restrict this to the domain of all each function.

The Odd Even Identities:

Note that because of the way we've defined Sine and Cosine:

$\sin(-\theta) = -\sin(\theta)$ An ODD function.

$\cos(-\theta) = \cos(\theta)$ An EVEN function.

Note that a function can be neither ODD or EVEN.

$$f(x) = x + 1$$

Example 2: Show that the tangent function is an ODD function:

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} - \text{Definition of Tangent}$$

$$\frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} - \text{Odd/Even Functions}$$

$$\frac{-\sin(\theta)}{\cos(\theta)} = -\frac{\sin(\theta)}{\cos(\theta)} - \text{Associative Property of Multiplication}$$

$$-\frac{\sin(\theta)}{\cos(\theta)} = -\tan(\theta) - \text{Definition of Tangent}$$

Other Even/Odd identities

$$\sec(-\theta) = \sec(\theta)$$

$$\csc(-\theta) = -\csc(\theta)$$

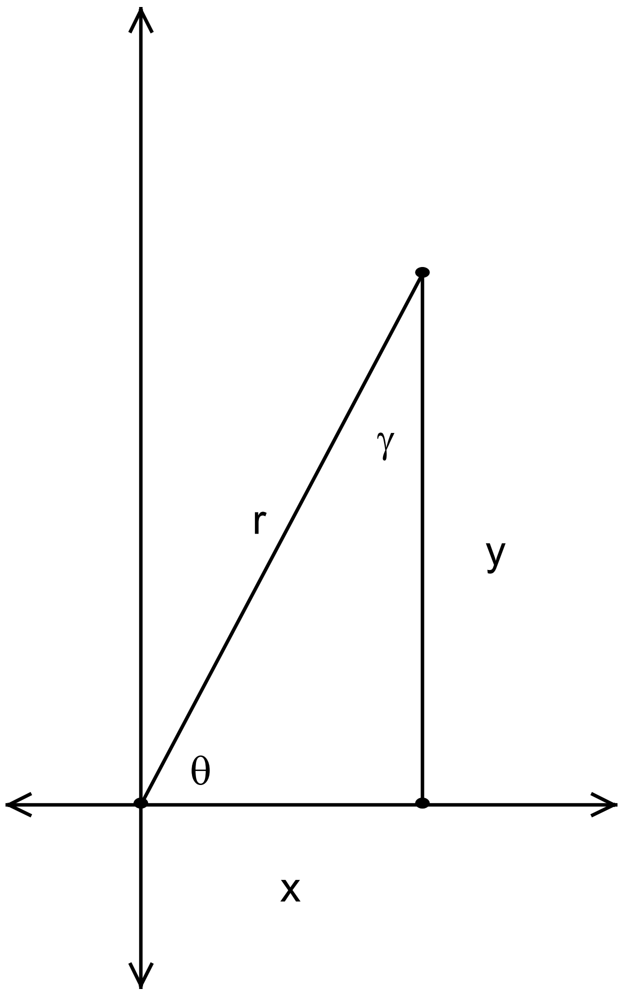
$$ctn(-\theta) = -ctn(\theta)$$

Example:

Find solutions to $\tan(-\theta)\csc(\theta) = 1.4$ over $[0, 2\pi]$

Find solutions to $\dfrac{3\sin(-\theta)}{\sin(\theta)} = 5\cot(\theta)$ over $[0, 2\pi]$

Co Function Identities



Note that

$$\sin\left(\theta\right)=\frac{y}{r} \qquad \cos\left(\gamma\right)=\frac{y}{r}$$

also

$$\cos\left(\theta\right)=\frac{x}{r} \qquad \sin\left(\gamma\right)=\frac{x}{r}$$

Since θ and γ are complementary, $\theta = 90^\circ - \gamma$

So we have the "co-function" identities

$$\sin\left(\theta\right) = \cos\left(90^\circ - \theta\right)$$

$$\cos\left(\theta\right) = \sin\left(90^\circ - \theta\right)$$

Looking at the reciprocal of these identities you get

$$\csc\left(\theta\right) = \sec\left(90^\circ - \theta\right)$$

and

$$\sec\left(\theta\right) = \csc\left(90^\circ - \theta\right)$$

and now look at the tangent function:

$$\tan\left(\theta\right)=\frac{\sin\left(\theta\right)}{\cos\left(\theta\right)}=\frac{\cos\left(90^{\circ}-\theta\right)}{\sin\left(90^{\circ}-\theta\right)}=ctn\left(90^{\circ}-\theta\right)$$

or

$$\tan\left(\theta\right)=ctn\left(90^{\circ}-\theta\right)$$

and

$$ctn\left(\theta\right)=\tan\left(90^{\circ}-\theta\right)$$

Examples:

Find all solutions to $\cos\left(\frac{\pi}{2}-\theta\right)=0.8$

Example:

Simplify $\cot\left(\frac{\pi}{2}-\theta\right)\sec\left(\frac{\pi}{2}-\theta\right)$