Lesson Plan 12 Polar Coordinates Math 48C Mitchell Schoenbrun

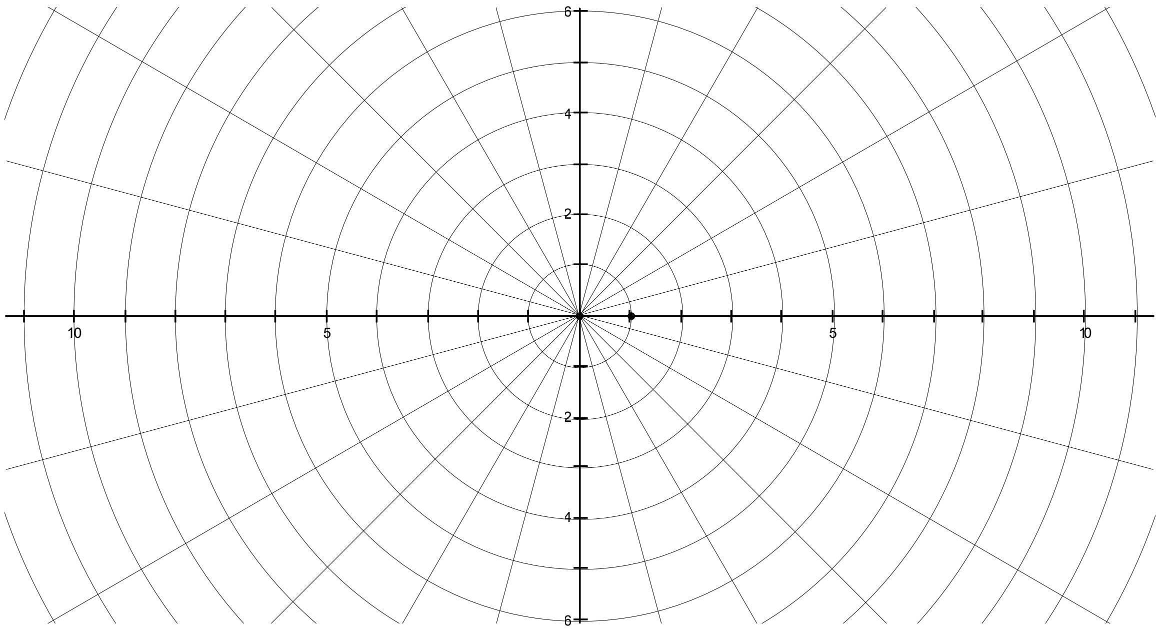
1) Attendance
2) Homework

Polar Coordinates.

What are Polar Coordinates?

Instead of X and Y coordinates
r and θ

Where *r* is the distance from the origin, and θ is the angle starting at the *x*-axis.

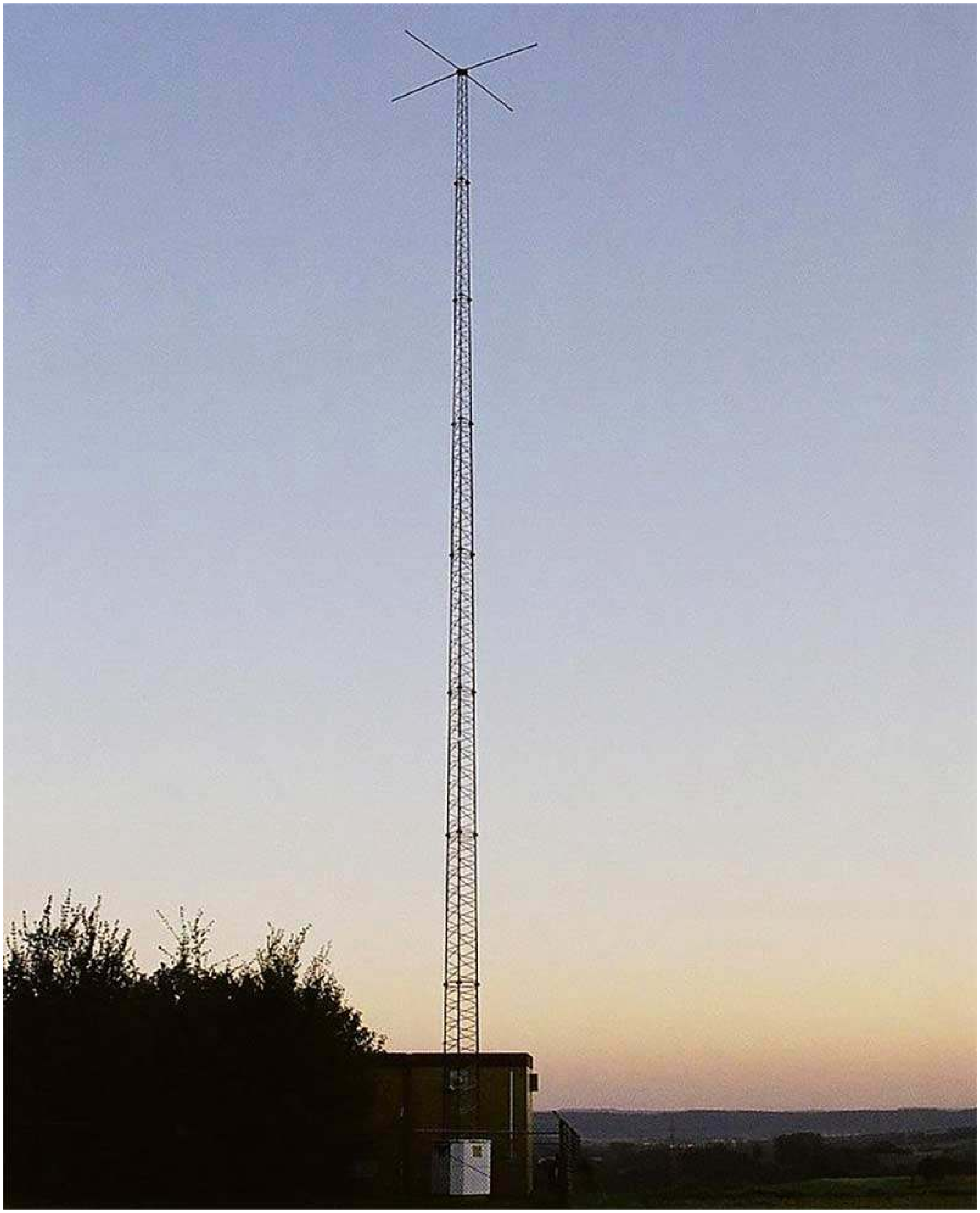


Each point on the graph has two coordinates (r,θ) instead of (x,y)

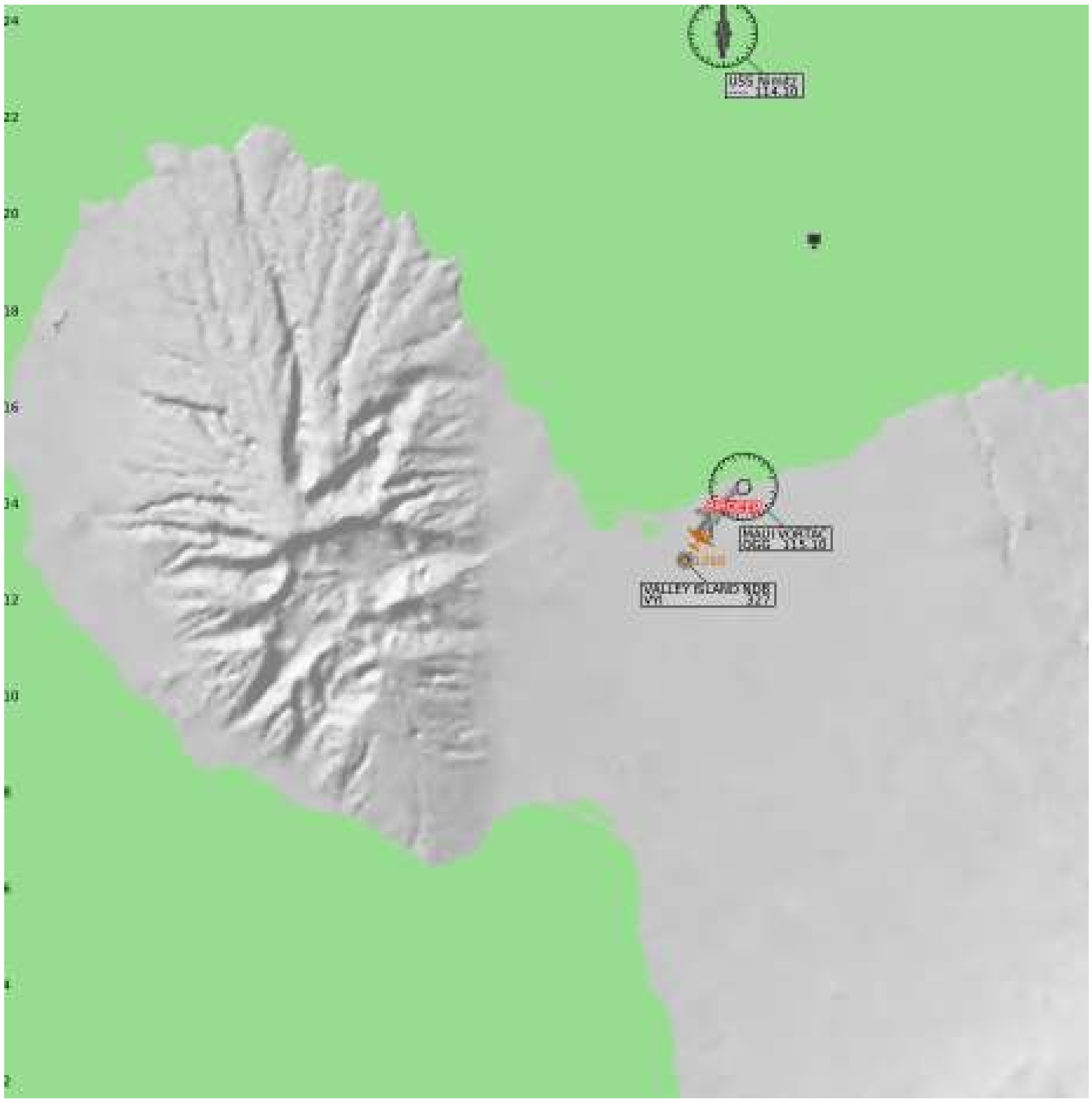
Applications of Polar Coordinates

1) Aircraft Navigation

NDB (Non-Directional Beacon).



Map of the Maui Airport showing an NDB location



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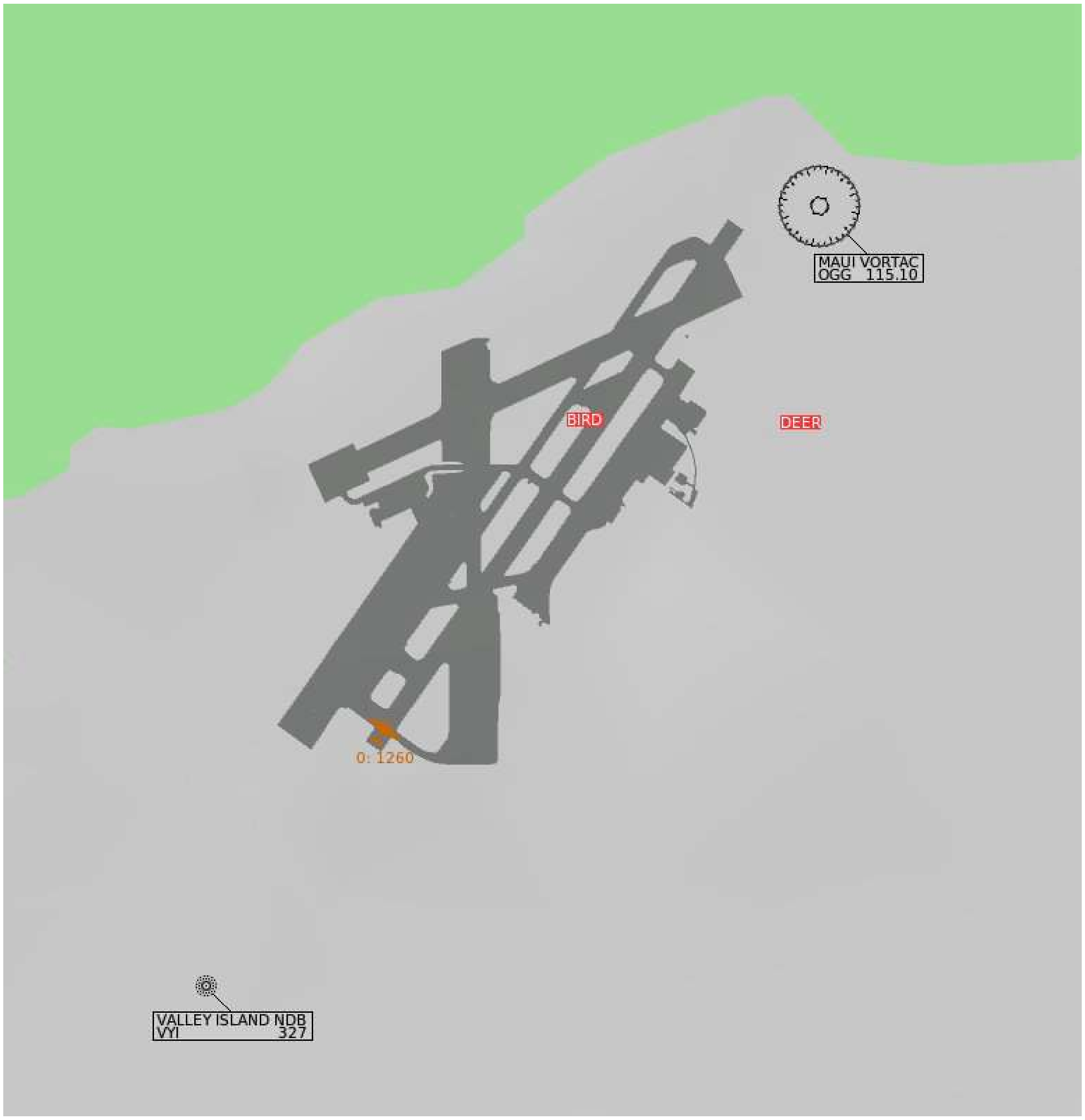
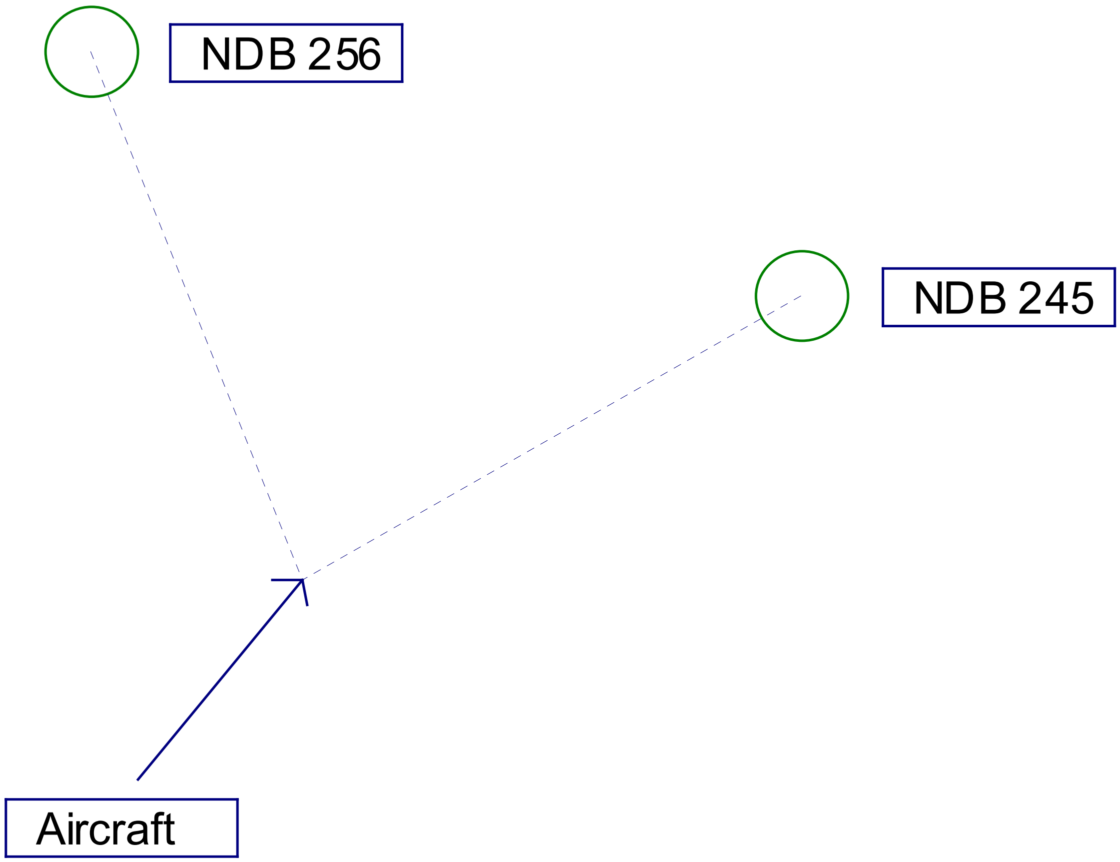


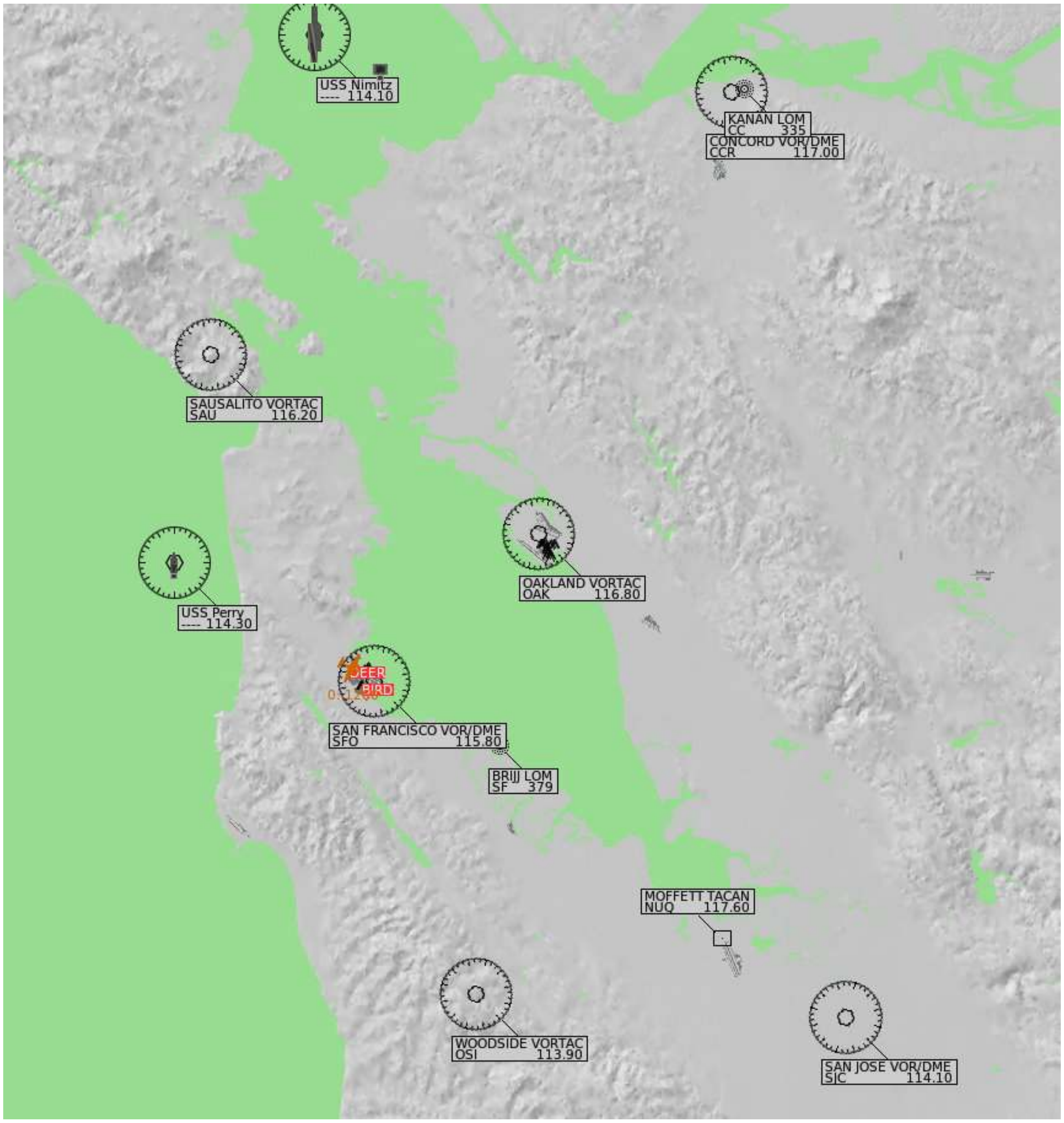
NDB's can also be used for triangulation if you have two of them.

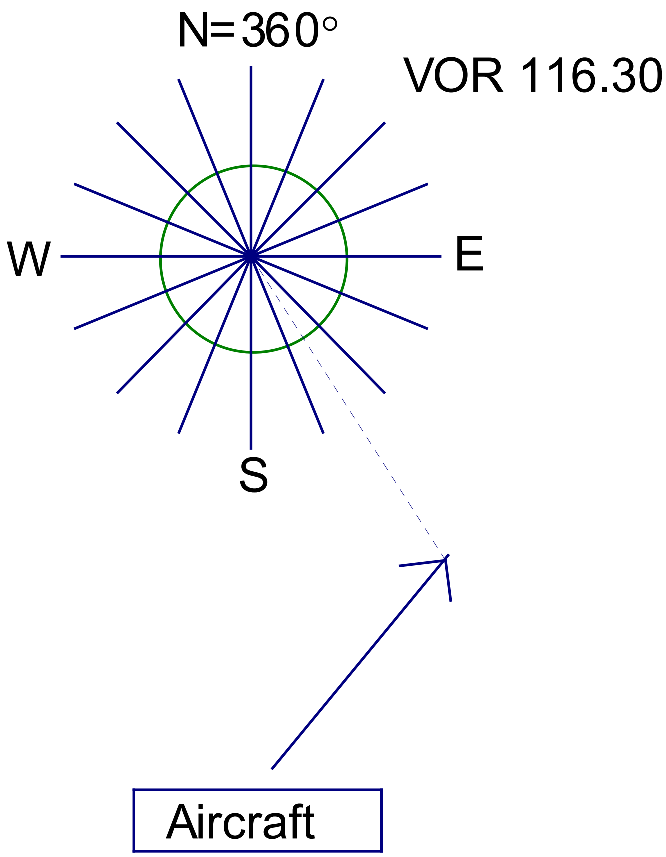


VOR   (VHF Omni-directional Range)

Gives more information about the angle, and sometimes also provides range (*r*) information.

San Francisco Bay Area VOR's




N=360°
VOR 116.30
W
E
S
Aircraft

Picture from X-10 Flight Simulator

Gauges are

Speed - Artificial Horizon - Altitude

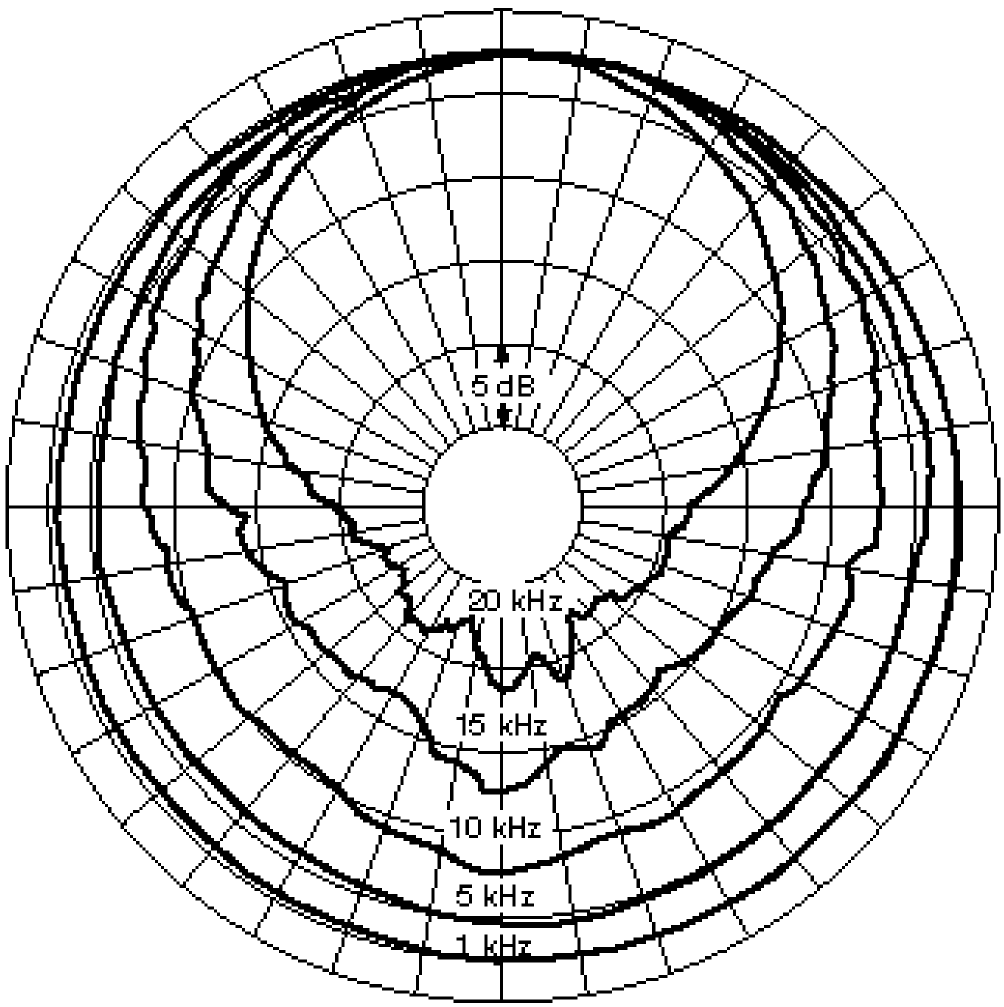
Yaw - VOR - Navigation - Vertical speed

NDB

MicroPhone Techonology

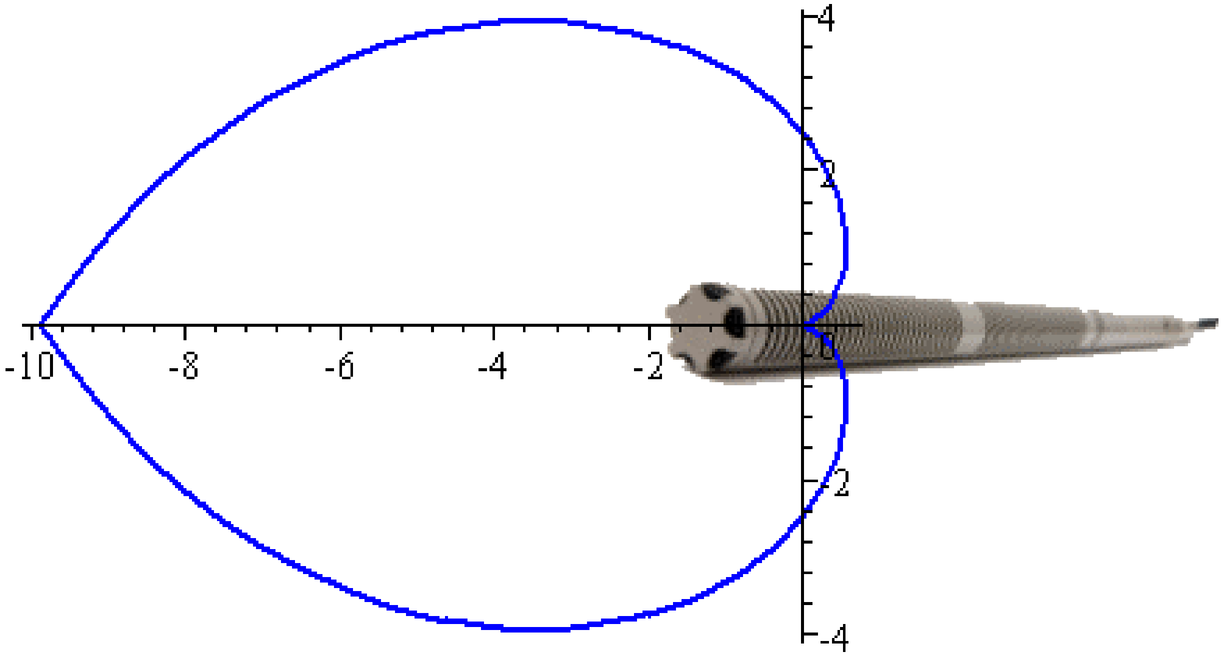
Microphones technology is concerned with the directionality of a Mic

CARTOID MIC - picks up sound directly in front of the mic.
Here the sound at different frequencies is mapped using polar coordinates.
We will look at the Cartoid curve later.

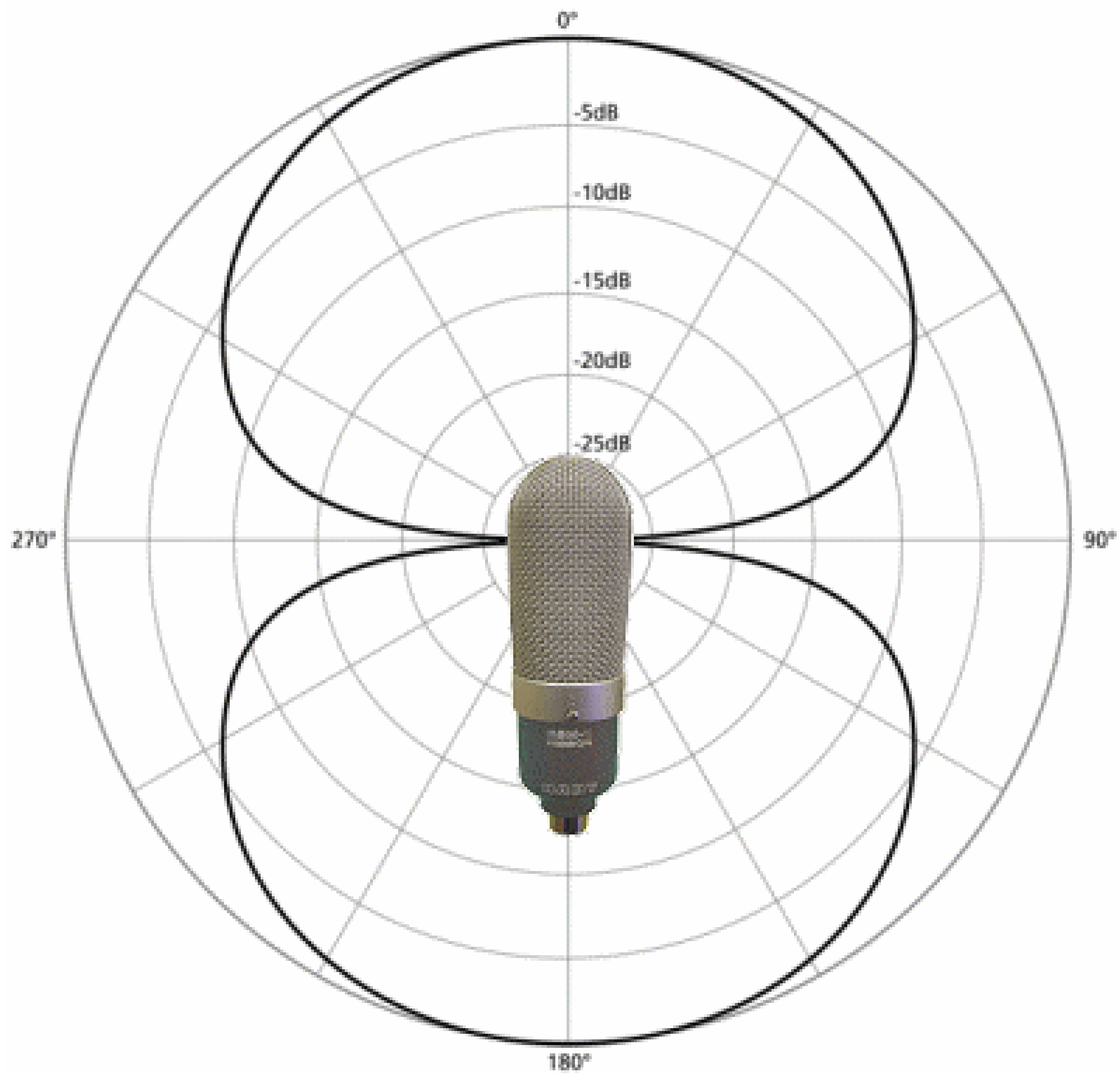


SHOT-GUN MIC - is highly directional.

This is also close to a Cartoid (Heart) shape.



A bi-directional mic used for interviews.



And of course maps of the polar regions can use polar coordinates.

Polar coordinates are not unique, for example

$$(1,0)=(1,2\pi)$$

$$(0,1)=(0,2)$$

(explain)

What is a polar equation?

An equation involving r and θ.
Usually θ is the independent variable and r is the dependent variable.

Examples: (Show using Grapher and TI-83)

1) $r=5$

A circle

2) r=θ (Archemedian Spiral)

$$r = a + b\theta$$

3) $r = \cos(\theta)$ (circle)

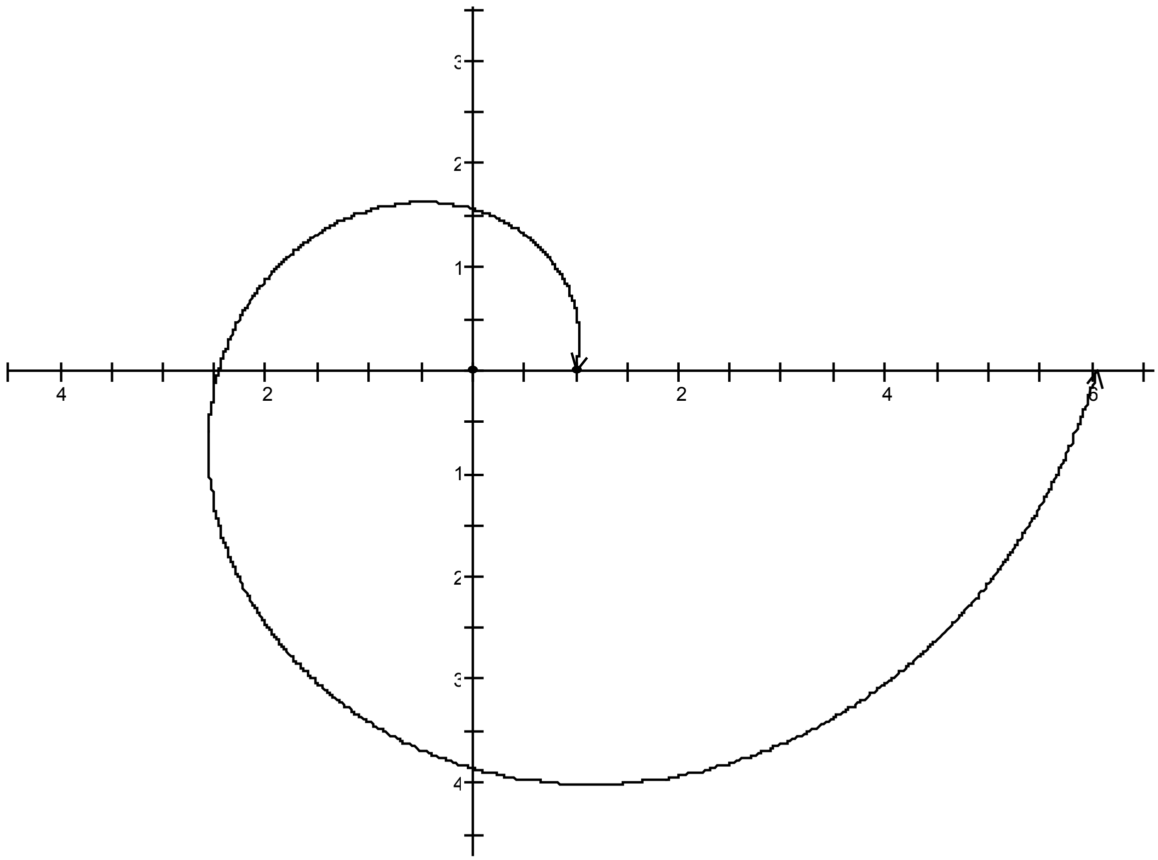
4) $r = \cos(2\theta)$ (clover loop)

5) $r = \cos(\theta/2)$ (folium)

6) $r = \cos(\theta/3)$ (folium)

7) $r = e^{\theta/8}$

Exponential spiral, one that appears quite often in nature.



A Nautilus Shell

A little curious history:

Jakob Bernoulli wanted a logarithmic spiral on his grave stone, but this is what he got.

It's an Archimedean Spiral

So who was Jakob Bernoulli? He came from a family of well known Mathematicians and scientists:

Daniel Bernoulli 1700-1782 came up with the famous Bernoulli principle.

Jakob Bernoulli, his uncle is the one with the wrong spiral on his gravestone.

Jakob had two brothers Johann and Nicolaus who were also famous mathematicians.

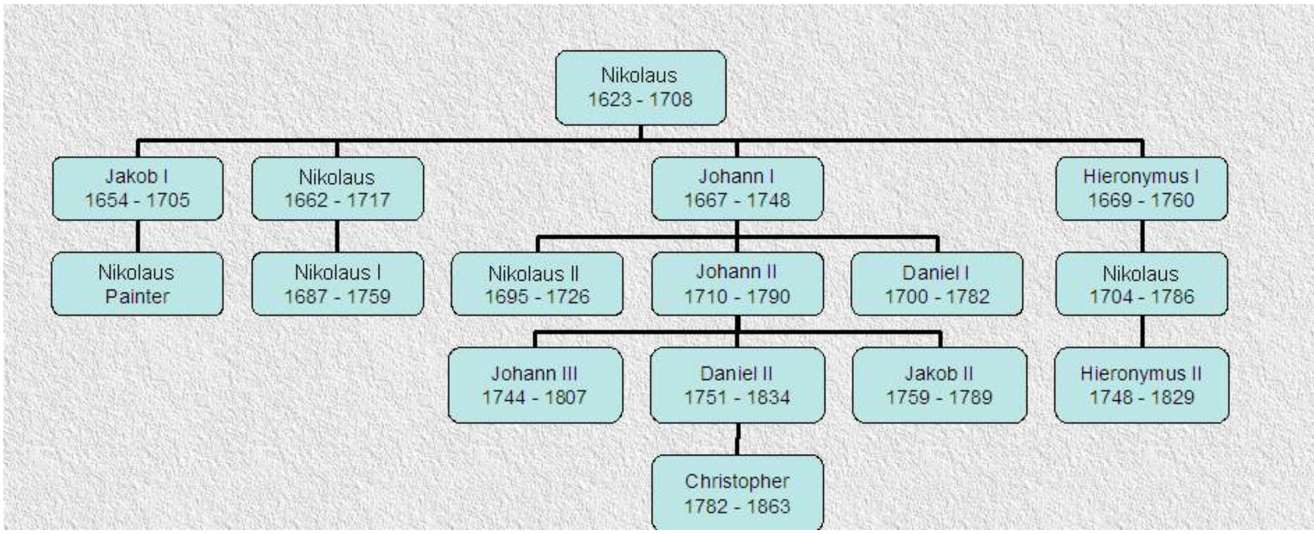
Daniel was Johann's son.

Daniel had a brother Johann II who was also a famous mathematician.

Johann II's son Johann the third was also a famous astronomer, geographer and mathematician.

Johann also had a son Jakob II who was a famous mathematician.

Here's a family tree.

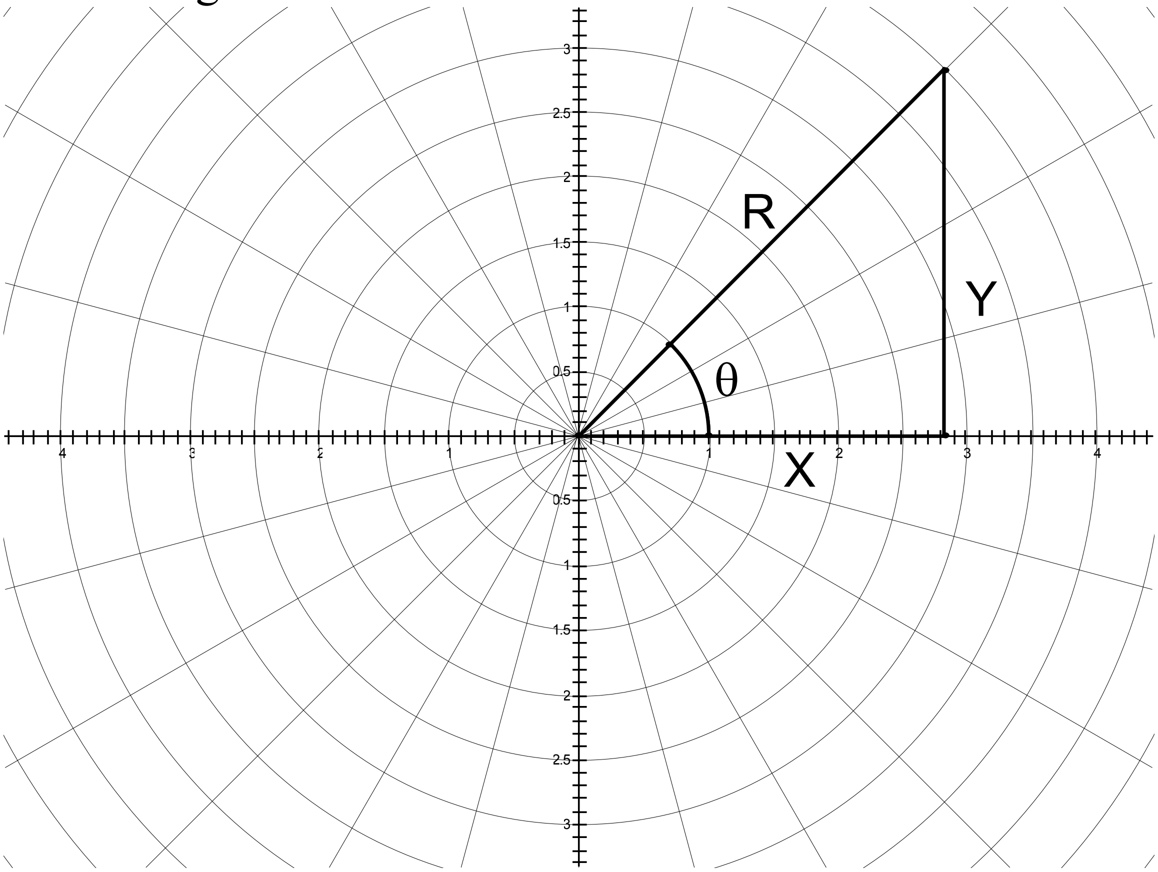


Makes you realize how little you and your family has done, :-(.

Use Grapher to show more examples.

Have students graph these on their calculator.

Converting from Polar to Cartesian Coordinates:



$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \text{ if } x \neq 0$$

if x=0 then $\theta = \frac{\pi}{2}$ or $\theta = \frac{3\pi}{2}$

Note there is some ambiguity as to which quadrant the point is in when using the $\tan^{-1}(\theta)$ function.

If result is positive, it could quadrant I or III. If negative II or IV.

This can be resolved by looking at the sign on *x* and *y*.

x > 0, y>0	I
x<0, y>0	II
x<0, y<0	III
x>0,y<0	IV

Note Many computer languages will have a multi-variable function

$$\tan 2(x, y)$$

which will compute the angle properly.

r=cos(theta*2)
r=cos(theta/2)
r=cos(theta/3)
r=1+cos(theta)

Demonstrate how to graph using calculator

Pass out some graph paper and have students graph r=1+cos(θ).

Demonstrate How to graph on a TI-83