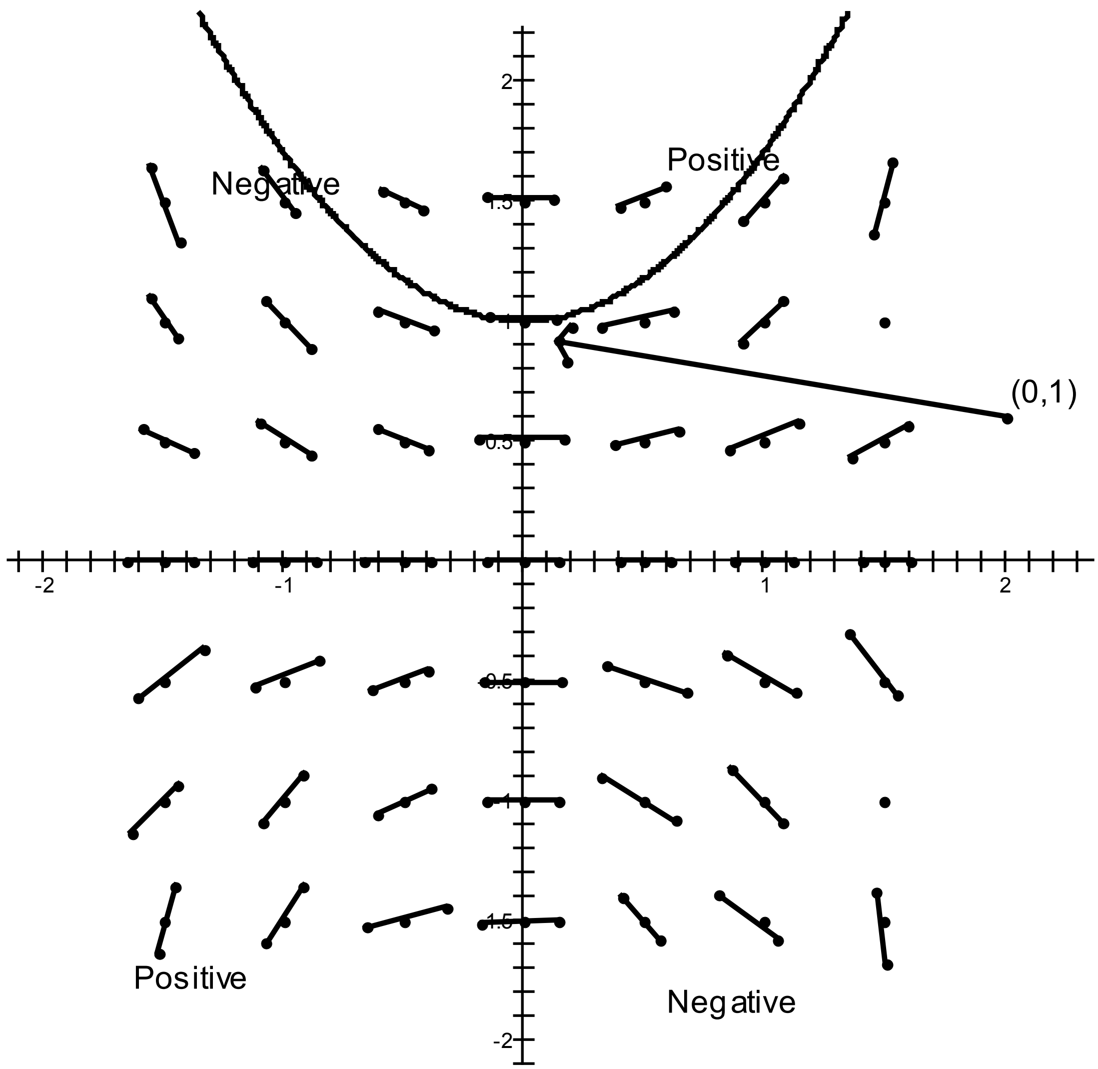
Foothill College Math 1B - Quiz 4 sections 7.1-7.3 Mitchell Schoenbrun

Name: Solution Key

1) Draw a direction field for the differential equation

$\frac{dy}{dx} = xy$ putting a line segment at each of the 49 points below.

Then draw an approximate solution starting at (0,1)



Notice that all points on the *x* and *y* axis have slope 0. Also note the symmetry of negative and positive slopes.

2) For the differential equation $y''-y=\sin x$, verify which of these possible solutions are correct.

a) $y=0$

b) $y=\cos x$

c) $y=-\frac{\cos x}{2}$

d) $y=-\frac{\sin x}{2}$

Keep in mind we are looking for a FUNCTION that is a solution

a) $y=0\rightarrow y'=0\rightarrow y''=0$

$0-0=0\neq\sin x$ [NOT A SOLUTION]

b) $y=\cos x\rightarrow y'=-\sin x\rightarrow y''=-\cos x$

$-\cos x-\cos x=-2\cos x\neq\sin x$ [NOT A SOLUTION]

c) $y=-\frac{\cos x}{2}\rightarrow y'=\frac{\sin x}{2}\rightarrow y''=\frac{\cos x}{2}$

$\frac{\cos x}{2}-\frac{-\cos x}{2}=\cos x\neq\sin x$ [NOT A SOLUTION]

d) $y=-\frac{\sin x}{2}\rightarrow y'=-\frac{\cos}{2}\rightarrow y''=\frac{\sin x}{2}$

$\frac{\sin x}{2}-\frac{-\sin x}{2}=\sin x$ [A SOLUTION!]

4) Find a solution to the differential equation $y' - 4y = 0$ with a starting point of $y(1) = 2$

$r - 4 = 0 \rightarrow r = 4 \rightarrow y(x) = Ae^{4x}$

$y(1) = 2 = Ae^{4} \rightarrow A = 2e^{-4}$

$y(x) = 2e^{-4}e^{4x} = 2e^{4(x-1)} = \frac{2}{e^{4}}e^{4x}$ [All correct solutions]

5) Find an implicit or explicit solution to the differential equation $\frac{dy}{dx}=\frac{x}{y-\cos y}$ with initial conditions $y(0)=4$ by separating variables.

Separating Variables:

$$(y-\cos y)\,dy = x\,dx$$
$$\int (y-\cos y)\,dx = \int x\,dx$$
$$\frac{y^2}{2}-\sin y=\frac{x^2}{2}+C$$ [This is an implicit solution which cannot be made explicit]

$$y(0)=4 \rightarrow \frac{4^2}{2}-\sin 4=\frac{0}{2}+C \rightarrow C=8-\sin 4$$
$$\frac{y^2}{2}-\sin y=\frac{x^2}{2}+8-\sin 4$$ [The Solution]

Extra Credit)
Write a differential equation for the orthogonal trajectory of the family of curves described by $\frac{x^2}{2}+\frac{y^2}{1}=k$

Differentiating implicitly

$$x+2y\frac{dy}{dx}=0 \rightarrow \frac{dy}{dx}=-\frac{x}{2y}$$

$\frac{dy}{dx}$ is the slope of the tangent at (x, y)

To be an orthogonal projection the slope must be perpendicular to this value, which is the negative reciprocal, so if we have

$\frac{dy}{dx}=\frac{2y}{x}$ we have the required differential equation.