Lesson Plan 9 - Improper Integrals 5.10

1) Take attendance
2) Questions before Quiz
3) Quiz
4) Review Approximate Integration

$\int_1^2 \frac{1}{x} dx$ letting n=5

Note the exact value is $A_{exact} = \ln(2) - \ln(1) = .693147$

$x_i = \{1.0, 1.25, 1.5, 1.75, 2.0\}$ so
$f(x_i) = \{1.0, 0.8, 0.66666667, 0.57142857, 0.5\}$

Using the website calculator we find that

$A_{left} = .75952381$
$A_{right} = .63452381$
$A_{trapezoid} = .69702381$
For the midpoint rule we can't use the website because it doesn't know the function:

$$A = \left(\frac{1}{4}\right)\left[\frac{1}{1.125} + \frac{1}{1.375} + \frac{1}{1.625} + \frac{1}{1.875}\right] = .6912198912$$

Left	Right	MidPoint	Trapezoid	Simpson	Exact
.75952381	.63452381	.6912198912	.69702381	.69325397	.6931471806
			.0039		

Please break up into groups of 3 or 4 and using your calculators and the website, do problem 10 on page 411. Have at least one person check the function calculations.

Use (a) the trapezoidal reule, (b) the midpoint rule and (c) SImposon's rule to approximate the given integral with the specified value of n. (Round to 6 decimal places)

$$\int_0^3 \frac{dt}{1+t^2+t^4}$$

Note here: Assignment for Thursday P. 411, 3, 5, 7, 9, 10, 28

Improper Integrals

A requirement so far for evaluating definite integrals using the fundamental theorem of Calculus so far has been that the function be continuous and the limits of the integral are finite values.

We can get around this for functions with jump discontinuities by separating the integral into the sum of multiple integrals, divided at the discontinuity.

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

where c is the place with the jump discontinuity occurs.

There are other integrals that we can evaluate with a little more care and work.

One example is a definite integral that is defined to infinity.

$$\int_1^\infty \frac{1}{x^2}\,dx$$

Another example is a function with a discontinuity with a limit at infinity, or one with a vertical asymptote.

$$\int_2^5 \frac{1}{\sqrt{1-x}}\,dx$$

This are respectively called type 1 and type 2 by the book

Let's first deal with Type 1.

We should note that this type of integral might have a finite value or it might diverge to infinity just as the following sequences do:

$\sum \frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\ldots$ This converges to ?

on the other hand,

$\sum \frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots$ this sequence diverges. That is for any large number M, there is an N such that the sum of N terms of this sum is greater than M.

How to evaluate $\int_1^\infty \frac{1}{x^2}\,dx$?

First we write extract the following function: $F(a)=\int_1^a \frac{1}{x^2}\,dx$

We then evaluate the limit

$$\lim_{a\to\infty} F(x)=\lim_{a\to\infty}\int_1^a \frac{1}{x^2}\,dx=\lim_{a\to\infty}\left[-\frac{1}{x}\right]_1^a=\lim_{a\to\infty}\left[-\frac{1}{a}+1\right]=\lim_{a\to\infty}1-\lim_{a\to\infty}\frac{1}{a}=1-0=1$$

We define this limit as the value of the integral, and we call this a CONVERGENT integral.

Note that we cannot simply insert ∞ as a value.

Now look at this integral $\int_1^\infty \frac{1}{x}\,dx=\lim_{a\to\infty}\int_1^a \frac{1}{x}\,dx=\lim_{a\to\infty}\left[\ln x\right]_1^a=\lim_{a\to\infty}\left[\ln a-\ln 1\right]=\lim_{a\to\infty}\ln a$

But this expression has no limit. $\ln(x)$ grows larger than any finite value.

This is called a DIVERGENT integral.

Example 2: $\int_{-\infty}^0 xe^x dx=\lim_{t\to-\infty}\int_{-t}^0 xe^x dx$

$$f=x \quad g'=e^x$$
$$f'=1 \quad g=e^x$$
$$\lim_{t\to-\infty}\int_{-t}^0 xe^x dx=\lim_{t\to-\infty}\left[xe^x-\int e^x\right]_{-t}^0=\lim_{t\to-\infty}\left[xe^x-e^x\right]_{-t}^0=$$
$$\lim_{t\to-\infty}\left[-1-\left(te^t-e^t\right)\right]=\lim_{t\to-\infty}te^t+e^t-1$$

The 2nd and third term are 0 and -1, but to evaluate the first term we need L'Hospital's rule.

$$\lim_{t\to-\infty}te^t=\lim_{t\to-\infty}\frac{t}{e^{-t}}=\lim_{t\to-\infty}\frac{\frac{d}{dt}t}{\frac{d}{dt}e^{-t}}=\lim_{t\to-\infty}\frac{1}{-e^{-t}}=-e^t=0$$

So $\int_{-\infty}^0 xe^x dx=-1$

Let's try a few of these from the handout!