Lesson Plan 6 - Integration by Parts 5.6

1) Take attendance
2) Return Quiz and go over
3) Questions on Homework

Chapter 5.6 Integration by parts

We start with the product rule:

$$\left[f(x)g(x)\right]' = f'(x)g(x) + f(x)g'(x)$$

We integrate both sides giving

$$\int \left[f(x)g(x)\right]' = \int f'(x)g(x) + \int f(x)g'(x)$$

On the left integration and differentiation cancel

$$f(x)g(x) = \int f'(x)g(x) + \int f(x)g'(x)$$

Subtracting the first part of the sum we get

$$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)$$

So given a function broken up into a product $f(x)g'(x)$

we can transform it so that instead of integrating $f(x)g'(x)$

we integrate $f'(x)g(x)$

Whether this is a good idea or not depends on whether the 2nd function is easier to integrate than the first.

Example 1:

Find $\int xe^x dx$

We have a few choices here but thinking that $x' = 1$ is pretty simple, we set

$$f(x) = x \qquad g'(x) = e^x \quad \text{then}$$
$$f'(x) = 1 \qquad g(x) = e^x$$

So $\int xe^x dx = xe^x - \int 1 \cdot e^x dx = xe^x - e^x + C$

In this case it might be good to see how this works in reverse:

$$\frac{d}{dx} xe^x - e^x + C = \left(xe^x + e^x\right) - e^x + 0 = xe^x$$

Example 2:

$\int x \sin x \, dx$

Again removing the x by setting it to f(x) might work.

$$f(x) = x \qquad g'(x) = \sin x \quad \text{then}$$
$$f'(x) = 1 \qquad g(x) = -\cos x$$

So $\int x \sin x \, dx = -x \cos x - \int -\cos x \, dx = -x \cos x + \sin x$

Example 3:

$\int \ln x \, dx$ Here we want to get rid of $\ln x$.

$$f(x) = \ln x \qquad g'(x) = 1 \quad \text{then}$$
$$f'(x) = \frac{1}{x} \qquad g(x) = x$$

$$\int \ln x \, dx = x \ln x - \int \frac{x}{x} dx = x \ln x - x + C$$

Example 3:

$$\int \frac{x^3}{\left(1+x^2\right)^3}\,dx$$

Note that $\dfrac{d}{dx}\dfrac{1}{\left(1+x^2\right)^2}=\dfrac{-4x}{\left(1+x^2\right)^3}$, so let

$$f(x)=-\frac{x^2}{4} \qquad g'(x)=\frac{-4x}{\left(1+x^2\right)^3} \quad \text{then}$$

$$f'(x)=-\frac{x}{2} \qquad g(x)=\frac{1}{\left(1+x^2\right)^2}$$

$$\int \frac{x^3}{\left(1+x^2\right)^3}\,dx=-\frac{x^2}{4}\cdot\frac{1}{\left(1+x^2\right)^2}-\int\frac{-x}{2\left(1+x^2\right)^2}=\frac{-x^2}{4\left(1+x^2\right)^2}-\frac{1}{4\left(1+x^2\right)}+C$$

Example 4: Definite Integration

$$\int_0^1 \tan^{-1} x\,dx$$

$$f(x)=\tan^{-1} x \qquad g'(x)=1 \quad \text{then}$$

$$f'(x)=\frac{1}{1+x^2} \qquad g(x)=x$$

$$\int_0^1 \tan^{-1} x\,dx=\left[x\tan^{-1}x-\int\frac{x}{1+x^2}\,dx\right]_0^1=\left[x\tan^{-1}x-\frac{1}{2}\ln\left(1+x^2\right)\right]_0^1=$$

$$\left[x\tan^{-1}x-\frac{1}{2}\ln\left(1+x^2\right)\right]_0^1=\tan^{-1}1-\frac{1}{2}\ln 2-0+\frac{1}{2}\ln 1=\frac{\pi}{4}-\frac{\ln 2}{2}$$

Example 5: Repeated Integration by parts

$\int x^2 e^x \, dx$

$f(x) = x^2 \qquad g'(x) = e^x$ then

$f'(x) = 2x \qquad g(x) = e^x$

$\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx$

$\int xe^x dx$

$f(x) = x \qquad g'(x) = e^x$ then

$f'(x) = x \qquad g(x) = e^x$

$\int xe^x dx = xe^x - \int e^x dx = xe^x - e^x = e^x(x-1)$

$\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx = x^2 e^x - 2e^x(x-1) = e^x(x^2 - 2x + 2)$

Pass out Handout 6

Assign Homework 5.6 P. 397 3, 13, 15, 16, 17, 25, 26, 40