Lesson Plan 16 - Applications to Physics 6.6

1) Take attendance
2) Homework questions?

We are going to look at some applications to Physics. This is not a Physics course so some explanations will be brief and to the point.

Force vs. Energy vs. Work

FORCE
The units of force are mass x acceleration. The metric unit of force is the Newton named after Sir Isaac Newton. A Newton is a Kilogram Meter per Second Squared.

Note a very confusing difference between Kilograms and Pounds. A Kilogram is a mass so it is the same on the earth as on the moon. A pound on the other hand is a measure of force so a pound on the moon has more mass than a pound on the earth. To keep things from getting confusing, I won't use pounds.

As you all may know, force is a vector quantity, meaning it has a magnitude and a direction.

You should be aware of the equation $\vec{F} = m\vec{a}$
A specific force will cause a specific mass to accelerate at a rate of $\frac{\vec{F}}{m}$.

Note this is in the absence of any other forces.

Consider what happens when I push against the wall.

I am putting a force against the wall, but nothing is accelerating, why not?

Because the wall pushes back with equal force.

This is also why you don't fall through the floor.
Note the source of this repulsive force is the repulsion between the electrons in the atoms on the wall and your body.

ENERGY
Force is not the same thing as energy. Energy is not a vector, but instead is a scalar quantity.
Energy has the units of mass x velocity squared which is the same as force x distance.
The metric unit of Energy is the Joule, a Newton-Meter.

WORK

Work is the name given to the transfer of mechanical energy by a force over a distance. Work has the same units as energy.

Note that a force applied to a mass which does not move has no energy.

An example of work:

I put a heavy object on a frozen lake so that it moves without friction.
I push the object with a mass of 10 kilograms with a force of 100 newtons a distance of 10 meters. The object accelerates at a rate of

100 Newtons/10 kilograms = 10 meters/second^2 over the 10 meters.

Work = Force x Distance = 100 Newtons x 10 Meters = 1000 Joules.

Since the object has absorbed 1000 Joules is now moving at

Sqrt(1000 Joules/10 Kilograms) = 10 meters/second.

Another way to do work on an object would be to lift it. The force you are lifting against is the force of gravity 'g' which is approximate 10 Newtons.

Example:

You lift 100 Kilograms 10 meters. How much energy did this require?

100 x 10 = 1000 Joules.

Applying Integration:

So what if the force varies as a function of distance?

If we were to break the distance into pieces, we would get

$$W = \sum F(x)\Delta x$$

If F(x) is an integrable function we would get $\int_a^b F(x)\, dx$

Examples with a varying force:

When a particle is located x feet from the origin a force of $x^2 + 2x$ Newtons acts on it. How much work is done moving it from x=1 meter to x=3 meters?

$$W = \int_1^3 \left(x^2 + 2x\right) dx = \left[\frac{x^3}{3} + x^2\right]_1^3 = \frac{50}{3}$$

Work needed to stretch a spring.

Hooke's law can apply to a spring in the following way.

The spring applies a force in the opposite direction that it is pulled or pushed in a linear manner:

$$F(x) = -kx$$

A force of 40 N is required to hold a spring stretched from its natural length of 10 cm to a length of 15 cm. How much work is done stretching the spring from 15 to 18 cm?

First note that $F(5) = 40N = -k40cm$ so

$$k = -\frac{40N}{5cm} = -\frac{40N}{.05m} = -800N/m$$

The Work then is $W = \int_{.05}^{.08} F(x)\,dx = \int_{.05}^{.08} 800x\,dx = \left[400x^2\right]_{.05}^{.08} = 1.56$ Joules.

Work needed to pump out a conical tank of water.

A tank in the shape of an inverted circular cone is 10 m tall and filled wit water to a depth of 8 feet. The radius at the top is 4 meters. The density of water is 1000kg/m^3.

How much work does it take to pump the water to the top of the cone, thus emptying it.

The mass of circle of water in the cone is $\pi r^2 dy$ in m^3 times the density 1000.

If the origin is at the bottom of the cone, then $\frac{r}{y} = \frac{4}{10}$ so $r = \frac{2y}{5}$ and the volume is

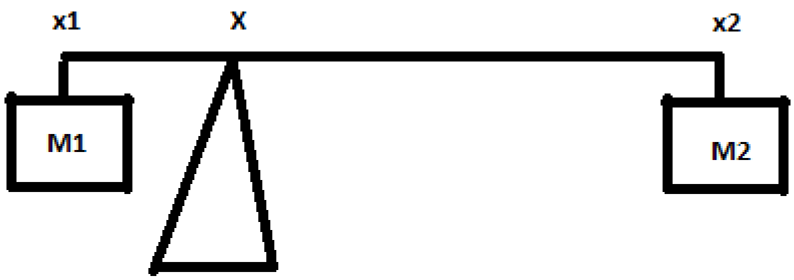
$\frac{4\pi}{25} y^2$ and therefore the mass is $160\pi y^2$

The distance each circle must rise is 10-y.

So the Work is $\int_0^8 160\pi y^2 \left(10 - y\right) g \, dy \approx 3.4x10^6$ Joules.

Moments and centers of mass.

If we have two weights at the ends of a bar balanced on a fulcrum as follows:



We have $d_2 = \bar{x} - x_1$ and $d_2 = x_2 - \bar{x}$

Then in order for the weights to be balanced, we need $m_1 d_1 = m_2 d_2$.

What we really need is for the apposing torques $m_1 g d_1 = m_2 g d_2$.

Substituting in we get $m_1\left(\bar{x} - x_1\right) = m_2\left(x_2 - \bar{x}\right)$ and therefore

$$m_1\bar{x} - m_1 x_1 = m_2 x_2 - m_2\bar{x}$$

$$m_1\bar{x} + m_2\bar{x} = m_1 x_1 + m_2 x_2$$

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Generalizing this to multiple weights we have:

$$\bar{x} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\sum m_i x_i}{m_t}$$

We call the value $M_x = \sum m_i x_i$ the "moment" of the system around the y-axis.

This is the place that an object spinning in space would rotate around.

If we find the intersection of the moments in x, y and z directions we have the center of mass or centroid.

In the case of a object whose mass in the *x* direction is *m(x)* we generalize this as follows:

$$\bar{x} = \frac{\int_a^b m(x)x\,dx}{\int_a^b m(x)\,dx} = \frac{\int_a^b m(x)x\,dx}{m}$$

If the density is a constant, then we can factor it out:

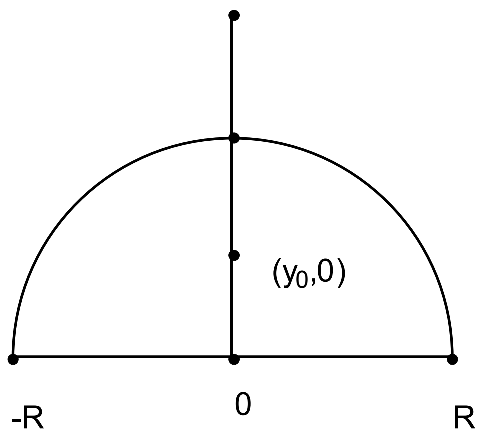
$$\bar{x} = \frac{\int_a^b m(x)x\,dx}{m} = \frac{\int_a^b \rho A(x)x\,dx}{pV} = \frac{1}{V}\int_a^b A(x)x\,dx$$

Or in the case of a flat plate object:

$$\frac{1}{A}\int_a^b f(x)x\,dx$$

Example 7 from the book.

We have a semi-circular plate of constant density as shown below:



Clearly in the *x* direction this is symmetric, so we just have to find the point along the *y* axis where it will balance. Call this point y_0.

So we need $\frac{1}{A}\int_a^b f(x)x\,dx = \frac{1}{\frac{1}{2}\pi R^2}\int_0^R (2x)\cdot y\,dy = \frac{4}{\pi R^2}\int_0^R\left(\sqrt{R^2-y^2}\right)y\,dy$

Solve $\int_0^R\left(\sqrt{R^2-y^2}\right)y\,dy$ using substitution:

$u = R^2 - y^2$
$du = -2y\,dy$

$$\int_0^R\left(\sqrt{R^2-y^2}\right)y\,dy = -\frac{1}{2}\int_0^R\left(\sqrt{R^2-y^2}\right)(-2y)\,dy = -\frac{1}{2}\int\sqrt{u}\,du = \frac{1}{2}\frac{(u)^{3/2}}{3/2} = \frac{\left[\left(R^2-y^2\right)^{3/2}\right]_0^R}{3} = \frac{R^3}{3}$$

So $\frac{4}{\pi R^2}\int_0^R\left(\sqrt{R^2-y^2}\right)y\,dy = \frac{4}{\pi R^2}\cdot\frac{R^3}{3} = \frac{4R}{3\pi} \approx .424R$