M1B/Schoenbrun Section 5.3: Fundamental Theorem of Calculus

Compute these integrals EXACTLY, check with your calculator

Find the indefinite integral or compute the definite integral EXACTLY

1) $\int_1^4 \sqrt{x}\,dx = \int_1^4 (x)^{1/2}dx = \left[\frac{x^{3/2}}{3/2}\right]_1^4 = \frac{2}{3}[8-1] = \frac{14}{3}$

2) $\int_{-2}^{3} \left|(x+1)(x-1)\right| dx =$

$$\int_{-2}^{-1}(x^2-1)\,dx - \int_{-1}^{1}(x^2-1)\,dx + \int_{1}^{3}(x^2-1)\,dx =$$

$$\left[\frac{x^3}{3}-x\right]_{-2}^{-1} - \left[\frac{x^3}{3}-x\right]_{-1}^{1} + \left[\frac{x^3}{3}-x\right]_{1}^{3} =$$

$$-\frac{1}{3} - {}^{-}1 - \left(\frac{-8}{3} - {}^{-}2\right) - \left[\frac{1}{3} - 1 - \left(-\frac{1}{3} - {}^{-}1\right)\right] + \frac{27}{3} - 3 - \left(\frac{1}{3} - 1\right) =$$

$$-\frac{1}{3} + 1 + \frac{8}{3} - 2 - \frac{1}{3} + 1 - \frac{1}{3} + 1 + 9 - 3 - \frac{1}{3} + 1 = \frac{4}{3} + 8 = \frac{28}{3}$$

3) $\int \frac{\cos(x)}{\sin(x)^5}\,dx = \frac{-1}{4\sin(x)^4} + C$

4) $\int \frac{t^5 - 4t^3 + t^2 - 8t + 5}{t^2}\,dt = \int t^3 - 4t - \frac{8}{t} + \frac{5}{t^2}\,dt = \frac{t^4}{4} - 2t^2 - 8\ln|t| - \frac{5}{3t^3} + C$

5) $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx = \arcsin\left(e^x\right) + C$

Find the derivative of each function

6) $f(x)=\int_1^2 x\,dx$ $\quad \frac{d}{dx}f(x)=0$ (Note that a definite integral is a number)

7) $g(x)=\int_1^x \frac{1}{t^3+1}dt$ $\quad \frac{d}{dx}g(x)=\frac{1}{x^3+1}$

8) $g(x)=\int_1^x e^{t^2-t}dt$ $\quad \frac{d}{dx}g(x)=e^{x^2-x}$

9) $F(x)=\int_x^\pi \sqrt{1+\sec t}\;dt=-\int_\pi^x \sqrt{1+\sec t}\;dt$ $\quad \frac{d}{dx}F(x)=-\sqrt{1+\sec x}$

13) $h(x)=\int_2^{1/x} \arctan(t)\,dt$ $\quad \frac{d}{dx}h(x)=\arctan\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)$

14) $h(x)=\int_0^{x^2} \sqrt{1+r^3}\;dr$ $\quad \frac{d}{dx}h(x)=\sqrt{1+x^6}\cdot 2x$

15) $y=\int_0^{\tan x} \sqrt{t+\sqrt{t}}\;dt$ $\quad \frac{dy}{dx}=\sqrt{\tan x+\sqrt{\tan x}}\,\sec^2 x$

16) $y=\int_{e^x}^0 \sin^3(t)\,dt=-\int_0^{e^x} \sin^3(t)\,dt$ $\quad \frac{dy}{dx}=-\sin^3\left(e^x\right)\cdot e^x$

17) $y=\int_{\sin x}^{\cos x} \left(1+v^2\right)^{10}\,dv=\int_{\sin x}^{0} \left(1+v^2\right)^{10}\,dv+\int_0^{\cos x} \left(1+v^2\right)^{10}\,dv=-\int_0^{\sin x} \left(1+v^2\right)^{10}\,dv+\int_0^{\cos x} \left(1+v^2\right)^{10}\,dv$

$\frac{dy}{dx}=-\left(1+\sin^2 x\right)^{10}\cdot\cos x+\left(1+\cos^2 x\right)^{10}\left(-\sin x\right)$