Section 7.2, 7.3 Trigonometric integration

1)

$$\sin^4 x = \left(1-\cos^2 x\right)^2 = 1-2\cos^2 x+\cos^4 x =$$

$$1-2\left(\frac{\cos 2x+1}{2}\right)+\left(\frac{\cos 2x+1}{2}\right)^2 =$$

$$1-\cos 2x-1+\frac{\cos^2 2x+2\cos 2x+1}{4}=$$

$$\frac{1}{4}-\frac{\cos 2x}{2}+\frac{1+\cos 4x}{8}=\frac{3}{8}-\frac{\cos 2x}{1}+\frac{\cos 4x}{8}$$

$$\int \sin^4 x\,dx = \int \frac{3}{8}-\frac{\cos 2x}{2}+\frac{\cos 4x}{8}dx = \frac{3}{8}x-\frac{\sin 2x}{4}+\frac{\sin 4x}{32}+C$$

2)

$$\sin^5 x = \sin x\left(1-\cos^2 x\right)^2 = \sin x - 2\sin x\cos^2 x+\sin x\cos^4 x =$$

$$\int \sin^5 x\,dx = \int \sin x-2\sin x\cos^2 x+\sin x\cos^4 x\,dx =$$

$$-\cos x+\frac{2\cos^3 x}{3}+\frac{\cos^4 x}{5}+C$$

3)

$$\sin^2 x\cos^3 x = \sin^2 x\left(1-\sin^2 x\right)\cos x = \sin^2 x\cos x-\sin^4 x\cos x$$

$$\int \sin^2 x\cos^3 x\,dx = \int \sin^2 x\cos x-\sin^4 x\cos x\,dx =$$

$$\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$$

4) There are a couple ways to do this integral, none of which I like or think I've prepared you for. The easiest is probably to substitute $x = 5\tan u$ which leaves you with the integral $\int \sec^3 u\,du$ to solve. This can be done using reduction formula #4 on page 526 and then using the table at the end of book to evaluate $\int \sec x\,dx$.

My preferred method would be to substitute $x = \sinh u$ which leaves you with the job of solving $\int \cosh^2 u\,du$. Since we haven't covered Hyperbolic functions yet, I cannot recommend this approach.