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15)

$$\int \sin^2 x \cos^2 x \, dx = \int \frac{(2\sin x \cos x)^2}{4} dx = \int \frac{\sin^2 2x}{4} dx =$$

$$\frac{1}{4}\int \frac{\cos 4x + 1}{2} dx = \frac{1}{4}\left[\frac{\sin 4x}{8} + \frac{x}{2}\right] + C = \frac{\sin 4x}{32} + \frac{x}{8} + C$$

17)

$$\int \sin^3 x \cos^2 x \, dx = \int \sin x \left(1 - \cos^2 x\right)\cos^2 x \, dx =$$

$$\int \sin x \cos^2 x - \sin x \cos^4 x \, dx = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$

25) $\int \tan^2 x \, dx = \int 1 - \sec^2 x \, dx = x - \tan x + C$

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7)

$$\int_0^{5/2} \frac{dx}{\sqrt{25-x^2}}$$

$$x = 5\sin u$$

$$dx = 5\cos u\, du$$

$$0 = 5\sin u \to u = 0$$

$$5/2 = 5\sin u \to u = \pi/6$$

$$\int_0^{5/2} \frac{dx}{\sqrt{25-x^2}} = \int_0^{\pi/6} \frac{5\cos u\, du}{\sqrt{25-25\sin^2 u}} = \int_0^{\pi/6} du = u\Big|_0^{\pi/6} = \pi/6$$

17)

$$\int \sqrt{64-x^2}\, dx$$

$$x = 8\sin u$$

$$dx = 8\cos u\, du$$

$$\int \sqrt{64-x^2}\, dx = \int \sqrt{64-64\sin^2 u} \cdot 8\cos u\, du = 64\int \cos^2 u\, du =$$

$$64\int \frac{\cos 2u+1}{2}\, du = 32\int \cos 2u + 1\, du = 32\left[\frac{\sin 2u}{2} + u\right] + C$$

$$\frac{x}{8} = \sin u$$

$$\cos u = \sqrt{1-\frac{x^2}{64}} = \frac{\sqrt{64-x^2}}{8}$$

$$u = \sin^{-1}\frac{x}{8}$$

$$32\left[\frac{\sin 2u}{2} + u\right] + C = 32\sin u\cos u + 32u + C =$$

$$32\frac{x}{8} \cdot \frac{\sqrt{64-x^2}}{8} + 32\sin^{-1}\frac{x}{8} + C = \frac{x\sqrt{64-x^2}}{2} + 32\sin^{-1}\frac{x}{8} + C$$

31)

$$\int \frac{\sqrt{x^2-9}}{x}\,dx$$

$$x = 3\sec u$$

$$dx = 3\sec u \tan u\,du$$

$$\int \frac{\sqrt{x^2-9}}{x}\,dx = \int \frac{\sqrt{9\sec^2 u - 9}}{3\sec u}\cdot 3\sec u \tan u\,du =$$

$$\int 3\sqrt{\sec^2 u - 1}\tan u\,du = 3\int \tan^2 u\,du = 3\int \sec^2 u - 1\,dx = 3\tan u - 3u + C$$

$$x = \frac{3}{\cos u} \to \cos u = \frac{3}{x} \to \sin u = \sqrt{1-\frac{9}{x^2}}$$

$$u = \cos^{-1}\frac{3}{x}$$

$$3\tan u - 3u + C = 3\frac{\sin u}{\cos u} - 3u + C = 3\frac{\sqrt{1-\frac{9}{x^2}}}{\frac{3}{x}} - 3\cos^{-1}\frac{3}{x} + C =$$

$$\sqrt{x^2-9} - 3\sec^{-1}\frac{x}{3} + C$$

50)

$$\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$$

$$x = 2\sin u$$

$$dx = 2\cos u\,du$$

$$\sin u = \frac{1}{2} \to u = \frac{\pi}{6}$$

$$\sin u = \frac{\sqrt{2}}{2} \to u = \frac{\pi}{4}$$

$$\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}} = \int_{\pi/6}^{\pi/4} \frac{2\cos u\,du}{4\sin^2 u\sqrt{4-\sin^2 u}} = \int_{\pi/6}^{\pi/4} \frac{2\cos u\,du}{4\sin^2 u\cdot 2\cos u} =$$

$$\frac{1}{4}\int_{\pi/6}^{\pi/4} \frac{du}{\sin^2 u} = \frac{1}{4}\int_{\pi/6}^{\pi/4} \csc^2 u\,du = -\frac{1}{4}\left[ctn\right]_{\pi/6}^{\pi/4} =$$

$$-\frac{1}{4}\left[\frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right] = -\frac{1}{4}\left[1-\sqrt{3}\right] = \frac{\sqrt{3}-1}{4}$$