# Solutions to old Final

1) $\frac{d}{dx}\int_1^{x^3}\cos(u)\,du=\cos\left(x^3\right)\cdot 2x^2$

2) C

3) 1.6289055

4) Let $u=e^x$ so $du=e^x dx$

$$\int\frac{e^x}{1+e^x}dx=\int\frac{du}{1+u}=\ln|1+u|+C=\ln\left|1+e^x\right|+C$$

5)

$$f=x \qquad g'=\sin x$$
$$f'=1 \qquad g=-\cos x$$

$$\int x\sin x=-x\cos x-\int-\cos x\,dx=-x\cos x+\sin x+C$$

6)

$$Avg=\int_0^2\frac{x^4-x^2}{2-0}dx=\frac{1}{2}\left(\frac{x^5}{5}-\frac{x^3}{3}\right)\Bigg|_0^2=\frac{1}{2}\left[\left(\frac{32}{5}-\frac{8}{3}\right)-0\right]=\frac{16}{5}-\frac{4}{3}=\frac{28}{15}$$

7)

$$\int_4^\infty e^{-x/2}dx=\lim_{a\to\infty}\int_4^a e^{-x/2}dx=\lim_{a\to\infty}\left[-2e^{-x/2}\right]_4^a=\lim_{a\to\infty}\left[-2e^{-a/2}-^{-}2e^{-2}\right]=$$

$$\lim_{a\to\infty}\left[-2e^{-a/2}\right]+2e^{-2}=\frac{2}{e^2}$$

8)

$$\int\frac{5x-4}{2x^2+x-1}dx=\int\frac{5x-4}{(2x-1)(x+1)}dx=\int\frac{A}{2x-1}+\frac{B}{x+1}dx$$

$$Ax+B+2Bx-B=5x-4$$
$$A+2B=5$$
$$A-B=-4$$
$$B=3$$
$$A=-1$$

$$\int\frac{A}{2x-1}+\frac{B}{x+1}dx=\int\frac{-1}{2x-1}+\frac{3}{x+1}dx=$$

$$-\ln|2x-1|+3\ln|x+1|=\ln\left|\frac{(x+1)^3}{2x-1}\right|$$

9)
at x=e g(x)=f(x) so

$$Area = \int_{1}^{e} \ln x + 4 - \frac{5x}{e} dx + \int_{e}^{4} \frac{5x}{e} - \ln x - 4 \; dx =$$

$$x \ln x - x + 4x - \frac{5x^2}{2e} \Big|_{1}^{e} + \frac{5x^2}{2e} - x \ln x + x - 4x \Big|_{e}^{4} =$$

$$3e + \frac{85}{2e} - 15 - 4\ln(4)$$

10)

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \rightarrow x^2 = 9\left(1 - \frac{y^2}{4}\right)$$

$$\frac{\int_{2/3}^{2} \pi x^2 \, dy}{\int_{-2}^{2/3} \pi x^2 \, dy} = \frac{\int_{2/3}^{2} 9\pi\left(1 - \frac{y^2}{4}\right) dy}{\int_{-2}^{2/2} 9\pi\left(1 - \frac{y^2}{4}\right) dy} = \frac{\int_{2/3}^{2} 1 - \frac{y^2}{4} dy}{\int_{-2}^{2/3} 1 - \frac{y^2}{4} dy} = \frac{y - \frac{y^3}{12}\Big|_{2/3}^{2}}{y - \frac{y^3}{12}\Big|_{-2}^{2/3}} = \frac{7}{20}$$

12)

$$\frac{dy}{dx} = xe^y \rightarrow \int e^{-y} dy = \int x \, dx \rightarrow -e^{-y} = \frac{x^2}{2} + C \rightarrow y(x) = \ln\left(C - \frac{x^2}{2}\right)$$

$$y(0) = 1 = \ln(C) \rightarrow C = e$$

$$y(x) = \ln\left(e - \frac{x^2}{2}\right)$$